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16. ABSTRACT

The report gives results of an evaluation of the possibility that plant efficiency losses or combustion instability might limit NOx reduction by combustion modification. Data from natural-gas- and oil-fired boilers were used in the analyses. The study of effects on plant efficiency of combustion modifications for NOx reduction showed that the effects were negligible, at least within the scatter of available data. Nearly all plant efficiency variations (losses of up to 6%) resulted from plant load variation, which is not considered a combustion modification for NOx control. Combustion instability, however, appeared to be a possible limitation, if not properly understood and accounted for by hardware modifications. Fuel-rich burner operation, in the staged combustion technique, can create an unstable air-side feed system coupled mode of combustion instability A method of analyzing such instability modes was developed for use in providing stable operating conditions, even with very fuel-rich burners.

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EFFECTS OF COMBUSTION MODIFICATIONS FOR NO_x CONTROL ON UTILITY BOILER EFFICIENCY AND COMBUSTION STABILITY

by

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PREFACE

This study is a continuation of previous Aerospace Corporation efforts reported in "Analysis of Test Data for NO_x Control in Gas- and Oil-Fired Utility Boilers" (EPA-650/2-75-012, January 1975) and in "Analysis of Test Data for NO_x Control in Coal-Fired Utility Boilers" (EPA-600/2-76-274, October 1976). The data published in the earlier report was used as a basis for the analyses reported herein.

This study, as well as the two previous studies, was conducted for the U.S. Environmental Protection Agency, Combustion Research Branch, Industrial Environmental Research Laboratory, Research Triangle Park, North Carolina, during the second year of a three-year continuing grant. (The first study was conducted under a separate EPA Grant No. R-802366 for this same EPA office.) The first two studies concerned the effects of combustion modifications on NO_x emissions, whereas the effort reported here concerns possible side effects of these combustion modifications on plant efficiency and combustion stability. In the third and final year of the current grant, all of the previous studies will be combined and simplified into an overall design model capable of indicating design modifications to minimize NO_x emissions while maintaining low emissions of other air pollutants, high plant efficiency and stable combustion.

A brief introduction is contained in Section I. Conclusions and recommendations are contained in Section II while Section III contains a brief summary of results. Section IV describes the analysis of the effects of combustion modifications on plant efficiency. Section V describes the development and application of a model of air-side feed system coupled modes of combustion instability in utility boilers.

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NOMENCLATURE

FORTRAN NOTATION

AFR = Air-fuel ratio, by weight.

AK1, AK2, AK3 = Intermediate constants specific to a given fuel, defined by Eqs. (A-12), (A-13), (A-14).

AK3L, AK3H = Intermediate constants specific to a given fuel and to the Low and High heats of combustion.

CES, CES1, CES2 = Constants in the derived function relating sensible heat losses to AFR and FLOAD. Eq. (A-24).

ESC = Efficiency data after correction for losses due to electrical equipment, to water vapor and sensible heat in the flue gases and to incomplete combustion.

LOAD (FLOAD) = Electrical load generated by the plant, megawatts (fraction of rated load).

NUM (DENOM) = Intermediate functions used to describe the numerator (denominator) of the complex function SMTD. Eqs. (A-3), (A-4), (A-5).

QLIC, QLH20, QLSEN = Heat losses due to incomplete combustion, to water vapor in the flue gases and to sensible heat in the flue gases, respectively. Eqs. (A-17), (A-18), (A-24).

QHHC (QHHCM) = High heat of combustion, including allowance for incomplete combustion (maximum high heat of combustion, with complete combustion).

SMTD = The total number of moles (dry) in the combustion products.

WFT = The total weight flow rate of fuel.

* = Designates the FORTRAN "multiply".

CHEMICAL COMPOUND NOTATION

C, H, O, N, S = Designates the moles (a, b, c, d, e, respectively) of atomic Carbon, Hydrogen, Oxygen, Nitrogen, and Sulfur, respectively, in a given fuel "molecule". Eq. (A-1).

CO, CO_2, (CO_{2th}), H_2O, N_2, NO, O_2, SO_2 = Designates molecular Carbon Monoxide, Carbon Dioxide (theoretical for complete combustion), Water, Nitrogen, Nitric Oxide, Oxygen, and Sulfur Dioxide, respectively.

a, b, c, d, e = Designate moles of each atomic species in a given fuel molecule. C, H, O, N, S, above and Eq. (A-1).

ARABIC NOTATION

A, B, C, D, E, F, G, H = Designates the moles of C, CO, CO_2, H_2O, O_2, N_2, NO, and SO_2, respectively, in the products of combustion. Eq. (A-1).

A′, B′, D′ = Intermediate constants in the development of the function for QLSEN. Eqs. (A-20), (A-21).

A_b = Cross-sectional flow area for air in a burner.

C = Capacitance within a burner. Eq. (B-21).

C_{pfg} = Specific heat at constant pressure of the boiler flue gases.

C_h = Fraction of combustion completed within a burner.

F_A, F_B, F_C, F_D, F_E, F_F, F_G = Functions used to simplify Eq. (B-73). Eqs. (B-78) through (B-84).

$F_a(i)$ (i = 1 - 5) = Arbitrary functions used to schematically designate the furnace pseudo-acoustics, in five of the six directions, in Figure 4.

F_b = A complex function partially describing the time-varying response of the total weight flow rate issuing from a burner to changes in the furnace pressure at the burner exit. Eqs. (B-39), (B-40), (B-41).

F_c, F_{fe}, F_{rr}, F_s = Functions used to simplify Eq. (B-73). Eqs. (B-85) through (B-88).

F_p = A complex function describing the time-varying response of the furnace pressure at a burner exit to changes in the total weight flow rate issuing from the burner. Eqs. (B-73) through (B-77).

$F(r)$ = A function describing changes in local furnace pressures resulting from changes in the local combustion air-fuel ratio (through changes in the combustion temperature and molecular weight of the combustion products).

F_1, F_2, F_3 = Functions used to simplify Eqs. (B-42), (B-43). Eqs. (B-46), (B-47), (B-48).

$K_i(K_{iavg})$ = Constant describing damping during acoustic wave travel and the efficiency of wave reflection at solid boundaries (an average of K_i in the six directions of wave travel). Eq. (B-67).

K_{ri} = The efficiency of reflection of acoustic waves at solid boundaries.

K_3 = An empirical coefficient relating the fraction of combustion completed within a burner (C_h) to the weight flow rate of air through the burner ($\dot{w}_b$). Eq. (1).

L = Inertance of the air within a burner.

L_b = Length of a burner in the flow direction.

L_h = Distance from a burner exit to a solid boundary, in all six directions.

$M(MW_f)$ = Molecular weight (of a fuel "molecule").

Mag = The magnitude of a complex function.

P = Pressure; furnace (P_f); furnace response (P_{fo}); furnace input, or driving pressure (P_{fi}); open loop response (P_o); open loop input, or driving pressure (P_i); constant windbox pressure ($\overline{P}_{wb}$); pressure at the exit of the boiler radiant section (P_{fe}); intermediate pressures in a burner (P_1, P_2, P_3); constant ambient pressure ($\overline{P}_{amb}$).

$\dot{Q}_s$ = Sensible heat leaving the boiler with the hot flue gases (time rate).

$\dot{Q}_{in}$ = Heat entering the boiler, in chemical form, in the fuel (time rate).

$\dot{Q}_{out}$ = Electrical energy, expressed in heat units, leaving the plant (time rate).

R = Flow resistance; inlet to a burner (R_i); burner exit region (due to presence of flame) (R_f); linearized R_i and R_f ($R_{i\ell}$, $R_{f\ell}$); linearized resistance to the flue gas flow from one burner in leaving the radiant section (exit) of the boiler (R_e); linearized resistance to the total flue gas flow leaving the radiant section of the boiler (R_{bp}); steady state resistance in the burner exit region due to the presence of a flame (R_3).

R_o = Universal gas constant.

Re, Im = The Real and Imaginary components of a complex function.

S = The LaPlace operator.

T = Temperature; of the flue gases entering the stack (leaving the air preheater) (T_s); ambient (T_{amb}).

T_1, T_2, T_3, T_4, T_{1a}, T_{3a} = Time constants in the expression for the dynamic response of a burner. Eqs. (B-29) through (B-33).

V_e = The control volume, at a burner exit, in which the volume flow rate and air-fuel ratio perturbations issuing from the burner are converted to furnace pressure perturbations.

X = The moles of air burning with one mole of fuel.

a = The acoustic velocity in the air within a burner.

c = The acoustic velocity in the furnace gases.

e = Designates an exponential term (natural).

g = The acceleration of gravity.

i = An index.

j = $\sqrt{-1}$

n = The exponent of the expression for the fraction of combustion completed within a burner. Eq. (1).

n_{bt} = The total number of burners in the boiler.

r = The weight air-fuel ratio; in the burner (r_b).

t = Time.

$\dot{w}$ = Weight flow rate; air at the burner inlet ($\dot{w}_i$); air leaving the burner ($\dot{w}_b$); fuel leaving the burner (constant) ($\overline{\dot{w}}_{fb}$); total flow in and out of the control volume in the furnace ($\dot{w}_{in}$, $\dot{w}_{out}$); total flow leaving the boiler radiant section ($\dot{w}_t$); total fuel flow rate into the boiler ($\dot{w}_f$).

w_s = Weight of gases stored in the control volume in the furnace.

GREEK NOTATION

$\Delta H_c (\Delta H_{ch})$ = The heat of combustion (the high heat).

$\Delta H^{\circ}_{ff} (\Delta H^{\circ}_{fw})$ = The heat of formation of the fuel (of water).

$\theta_b (\theta_p)$ = The phase angle in the response of burner flow to the driving furnace pressure (of furnace pressure to the driving burner flow).

τ = Time delay; from the burner exit to the region of concentrated combustion (combustion time delay) at the steady-state burner flow velocity (τ_c); for acoustic wave travel from a burner exit to the exit from the radiant section of the furnace (τ_e); for acoustic wave travel from a burner exit to and from a solid boundary, in the (i) direction (τ_i).

α = An acoustic wave damping coefficient in plane wave travel in the furnace. Eq. (B-67).

δ = Designates a dynamic perturbation quantity (infinitesimally small amplitude).

ω = Frequency, in radians per second.

SUPERSCRIPTS

$-$ = Time-invariant quantities.

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A special acknowledgment is also due, once again, to the Los Angeles Department of Water and Power (LA DWP) for its earlier cooperation in making available the data upon which this study is based. That data was originally released by LA DWP for The Aerospace Corporation study of the effects of combustion modifications on NO_x emissions from natural gas- and oil-fired utility boilers, and was published in EPA-650/2-75-012, January 1975.

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SECTION I

INTRODUCTION

Earlier studies (1), (2), and (3) by The Aerospace Corporation were oriented toward analyzing the effects of combustion modifications on NO_x emissions from natural gas-, oil-, and coal-fired utility boilers. Large samples of data from tests run on full-scale utility boilers for the purpose of empirically investigating these effects were used in the Aerospace analyses. Resulting data correlations, and such extrapolations which could be safely made, indicated no inherent limitations in reduction of NO_x emissions with the use of staged combustion modification techniques (i. e., burners out of service and/or NO_x ports). A question arose concerning what kinds of real, practical limitations might exist in application of these techniques and, therefore, what limitations might exist in reduction of NO_x emissions.

In the data sample used in the earlier study (1) of ten natural gas-and oil-fired utility boilers, potential limitations were observed involving possible excessive loss of plant efficiency and combustion instability. Among the data available to that study were measured levels of boiler electrical load and fuel flow rates. This enabled a gross calculation, from measured values, of an overall boiler or plant efficiency, defined as the dimensionless ratio of the electrical output of the plant to the heat input represented by the chemical energy of the fuel (in compatible units). This measure of efficiency reflects all of those losses which can be affected by combustion modifications as well as some which cannot (i. e., losses in the steam cycle). Accurate, measured data on coal flow rates were not available to the study of NO_x control in coal-fired boilers.

A preliminary, direct correlation of this overall plant efficiency against NO_x emissions indicated that, if indeed the efficiency was some direct function of NO_x emissions, efficiency losses of 7 to 10 percent could result from attempts to reduce NO_x emissions from the uncontrolled levels to 100 parts-per-million (ppm). Such a loss, of course, would make the NO_x reduction extremely expensive both in terms of cost and excessive energy requirements. It was not expected that such a direct relation existed between efficiency and NO_x emissions but it was not known whether the changes in hardware and/or operating conditions necessary to reduce NO_x would also unavoidably cause significant efficiency losses.

One task of this study, then, was to investigate the causes of the observed efficiency variations and to relate them to modifications necessary to reduce NO_x emissions. The end result of this task was to determine if combustion modifications made for the express purpose of NO_x reduction had any significant effect on plant efficiency.

A second observation from the data sample used in the study(1) was that certain staged combustion configurations exhibited high vibrations, or combustion instability, and/or flame liftoff (flames detaching from the burner exit and moving out into the furnace). The available data on combustion instability was investigated, to a limited extent, to evaluate possible mechanisms which might explain the instabilities. It was concluded that the vibrations resulted from a feedback coupling between furnace pressure and air flow through the burners (defined as an air-side feed system coupled mode of combustion instability). No effort was made in that study to investigate the phenomenon beyond this preliminary identification. It did appear, however, that these instabilities were associated with fuel-rich operation of the burners, in the staged-combustion technique for NO_x control. Thus, this type of problem could represent a significant limitation to maximum implementation of the staged-combustion technique and a real, practical limit on NO_x reduction. These instabilities were observed only in the data from natural gas-fired boilers. Although none were observed in the data from oil-fired boilers available to this study, such instabilities have occurred in other oil-fired boilers. No documentation on similar instabilities in coal-fired boilers was available.

A second task of this study, then, was to analyze such air-side feed system coupled modes of combustion instability in utility boilers of the types used to fire natural gas and oil fuels and to apply this analysis to the boilers in the available data sample. The end result of this task was to develop and verify a method of analysis which could be used by others to resolve specific cases and to determine if certain combustion modifications made for the express purpose of NO_x reduction lead inevitably to combustion instability.

Beyond the study just described, the third and final year of this grant will integrate and simplify the analyses for NO_x control, plant efficiency and combustion stability (and other data observations) conducted under a previous EPA grant (1) and the first two years of the current EPA grant. An example design application, for a new, oil-fired boiler, will be shown.

Other studies have indicated that erosion or corrosion of the boiler water walls (principally in coal-fired boilers) might also represent a limitation on the use of combustion modification techniques for NO_x control. This possibility is being experimentally investigated by other contractors and agencies (for example, Ref. 4).

SECTION II

CONCLUSIONS AND RECOMMENDATIONS

2.1 CONCLUSIONS

The major conclusions of this study can be summarized as follows:

a. Combustion modifications, made for the purpose of NO_x control, do not significantly affect overall plant efficiency.

b. Use of the staged-combustion technique for NO_x reduction, involving very fuel-rich first-stage combustion, results in a tendency toward unstable combustion and high boiler vibrations.

c. A method of analysis of such air-side feed system coupled modes of combustion instability was developed which can be used to provide stable operating conditions even with very fuel-rich first stage combustion.

Results of the evaluation of the effects of combustion modifications on plant efficiency showed that there are no significant losses which can be charged directly against these modifications. The majority of the efficiency variations observed in the data were attributed to load variations. Load variations are not considered combustion modifications for the express purpose of NO_x control. No significant efficiency losses could be charged against operation with burners out of service or with the use of NO_x ports. There was some indication that reduction in combustion air temperature, which would reduce NO_x emissions, might also significantly reduce plant efficiency, but the available data were not in a form that could be used to support such a conclusion.

A model and analysis technique were developed for air-side feed system coupled modes of combustion instability. The model was based on conventional analyses of such modes in the liquid rocket industry and, in the limit, reduced to such conventional rocket models. Comparison with limited data on instabilities in gas-fired boilers in the data sample (1) showed reasonably good agreement. This not only tended to verify the analysis but also to verify the conclusion (1) that the observed high boiler vibrations were indeed air-side feed system coupled modes of combustion instability. Further evaluation of such modes of instability showed that very fuel-rich burner operation, for the purpose of NO_x reduction, results in a tendency toward unstable combustion and high boiler vibrations. The evaluation also showed, however, that such instabilities can be controlled by recognition of the character of the problem and by proper design. Proper design for stability can include consideration of such design parameters as the flame anchoring technique and the burner air-side pressure drop. Combustion instability, therefore, is not considered to represent an inherent limit on NO_x reduction by combustion modification techniques but rather an undesirable side effect which must be accounted for by special design analyses and/or techniques.

2.2 RECOMMENDATIONS

Conclusions of this study relate to the effects of combustion modifications on two possible sources of limitation on NO_x reduction: excessive loss of plant efficiency and combustion instability. In general, neither area appears to represent inherent limits. Both evaluations, however, did reveal other problems which could represent such limits.

After reviewing the results of this and other studies, further investigations are recommended in the following areas:

a. Effects of combustion air temperature on plant efficiency

b. Effects of in-flame flue gas recirculation on plant efficiency

c. Effects of combustion modification techniques on the plant efficiency of coal-fired boilers

d. Burner designs which provide soundly stable flame anchoring in burners operated with very fuel-rich mixtures.

In the efficiency study there was some indication that reduction in combustion air temperature, sometimes considered as a means of reducing NO_x emissions, could significantly reduce plant efficiency. The data available to this study were not adequate to evaluate this possibility and further study is recommended. Also, combustion modification techniques other than those involving burners out of service and NO_x ports, such as in-flame flue gas recirculation, were not tested and their effects on plant efficiency were not studied.

In the combustion stability task, it was shown that some full-scale utility boilers are currently being operated with sufficient burners out of service (to control NO_x) that the remaining active burners are operating very near fuel-rich flammable limits. Not only is it important that each case be evaluated, and perhaps re-designed, to assure combustion stability, but additional study should be devoted to designing burners specifically to provide soundly stable flame anchoring when the overall burner air-fuel ratio is near or below the flammable limit. Without such design it is conceivable that the problem of providing a stable pilot flame, and reliable subsequent flame propagation, in a very fuel-rich burner could represent a real, practical limit to NO_x reduction by techniques involving burners out of service or NO_x ports.

SECTION III

SUMMARY

This study consisted of two tasks, both largely addressing potential limitations in utility boilers to NO_x reduction by combustion modification techniques. These limitations were related to possible excessive loss of plant efficiency and to combustion instability. Both tasks involved large samples of data obtained from natural gas- and oil-fired boilers which were reported previously by Dykema (1).

3.1 PLANT EFFICIENCY

The basic data used as a starting point in the study of efficiency were measured values of electrical load (plant output) and fuel flow rates (heat input), in compatible units. Efficiency losses and efficiency corrections were calculated and evaluated in six areas: a. electrical generating and control equipment; b. uncondensed water vapor in the flue gases; c. excess sensible heat entering the stack in the flue gases; d. incomplete combustion; e. variations in expansion efficiency through the steam turbine with varying boiler load; and f. an "other" category remaining after the overall plant efficiency data were corrected for the first five, identifiable losses.

Losses due to the electrical equipment and to uncondensed water vapor are clearly not functions of any combustion modifications for NO_x control. As a result, only a simple correction was derived. The efficiency data were corrected to remove these effects so that the remaining data would be more sensitive to the effects of combustion modifications, if any.

Combustion modifications could affect the excess sensible heat loss. A calculation for these losses was derived and evaluated over the range of operating variables represented in the data. It was concluded that efficiency losses from this source greater than about one-half percent would be reflected in increased combustion air temperatures. Evaluation of measured combustion air temperatures showed no significant differences between the nominal and modified operating conditions. As a result, it was concluded that combustion modifications do not significantly affect sensible heat losses. Again, these losses were calculated and the remaining efficiency data corrected to remove variations due to sensible heat.

Losses due to incomplete combustion were thought to be the most likely to increase with combustion modifications. Strongly fuel-rich combustion in the first stage could result in a number of energetic hydrocarbons, carbon monoxide (CO), and increased carbon loss with the particulate emissions. The measure of this loss used here was the relation of the CO_2 concentrations measured in the flue gases to the theoretical CO_2 concentrations calculated from stoichiometry, at the overall boiler air-fuel ratio corresponding to the measured oxygen (O_2) level. Zero loss due to incomplete combustion would be indicated by CO_2 concentrations which contain all of the carbon in the fuel.

No evidence was found in either the direct measured levels of CO_2 or in a calculated efficiency correction (depending on the measured CO_2 level) to indicate that combustion modifications for the purpose of NO_x control affect losses due to incomplete combustion. Of course, this loss is one which is particularly susceptible to correction by the boiler operator. If high CO levels or smoke are observed, the boiler operator could change the level of excess air, or some other variable, to eliminate these malfunctions. Perhaps the correct conclusion here, then, might be that no evidence was found that combustion modifications for the purpose of NO_x formation affect losses due to incomplete combustion to the extent that they are not readily correctable by the boiler operator. Sensible heat losses, which vary with boiler excess air (if that is the parameter used by the boiler operator to eliminate high CO or smoke) would vary only slightly.

Variations in boiler load are necessary to satisfy the varying electrical demand. It is well known that NO_x concentrations in the flue gases usually decrease significantly with decreasing load. Two major variables which change with boiler load are the combustion air temperature and the expansion efficiency through the steam turbine. The former affects NO_x emissions and the latter affects plant efficiency. Thus it was expected that both NO_x and plant efficiency would decrease with boiler load. Although load variation is not considered a practical NO_x control technique, it was necessary both to derive an efficiency correction for load-related losses and to evaluate what fraction of the observed efficiency variations results from this parameter.

An empirical correlation of efficiency data showed the parabolic curve typical of turbine efficiency variations, with maximum efficiency at the most common operating condition, about 80 percent of rated load. Load reduction from 80 to 40 percent of rated resulted in an efficiency decrease of more than six percent. Thus, excluding data scatter, almost all of the plant efficiency variations observed in the available data sample are due to load variations, leaving little other variation which might be charged against combustion modifications made for the express purpose of NO_x control.

It is possible that the observed parabolic form of the efficiency data with load does not represent steam turbine efficiency variations alone.

It is then further possible that part of the loss at low loads could result from the reduced combustion air temperatures. If combustion air temperatures alone were reduced at fixed load, then, to reduce NO_x emissions, significant efficiency losses might result. This would be a case where the combustion modification desired, for the purpose of NO_x control, would significantly reduce plant efficiency. Unfortunately, the data available to this study were not adequate to delineate the separate effects of combustion air temperatures on both efficiency and NO_x emissions. The appropriate conclusion of this study is still that the observed (significant) efficiency variations with load are not the result of a combustion modification made (at least in the data of this study) for the express purpose of NO_x control.

Finally, after correcting the original overall plant efficiency data for the five losses discussed above, the remaining data variations were examined for any other losses which might result from combustion modifications. Although there was some indication of a direct relation of efficiency to the combustion temperature in the fuel-rich first stage, the effect could not be larger than about 0.8 percent. This variation is small compared to the data scatter. The final efficiency data, corrected for all of the losses discussed herein, had a standard deviation of 2.2 percent.

3.2 COMBUSTION STABILITY

Models and analysis techniques of feed system coupled modes of instability developed in the rocket industry were used as a basis for the development of a method of analysis for such feedback systems coupled to the air flow system in utility boilers. A major modification of those models and techniques was necessary to adequately describe the coupling between burner flow rate perturbations and resonances in the three coordinates of the furnace cavity. This modification greatly complicated the analysis but it was shown that, in the limit (primarily, in the case where the furnace cavity dimensions are very small), the analysis developed here becomes identical with that long used in the rocket industry.

Basically, the model can be described as follows. The boiler windbox is taken as a large, constant-pressure plenum from which combustion air enters the burners. The air in the burners has compressibility and inertia. Resistance to air flow through the burner is in two parts, a constant resistance at the burner inlet (through the air registers) and a variable resistance near the burner exit which is a function of the degree of initial combustion within the burner.

The pressure drop across the burner from the windbox to the furnace is very small, measured in inches of water. As a result, this pressure drop and the resulting air flow rates are quite sensitive to variations in furnace pressures at the burner exit. Perturbations in furnace pressure at the burner exit cause perturbations in the air flow rate through the burner.

The pressure drop across the fuel injectors (orifices) however is quite large, usually being measured in pounds per square inch (psi). Perturbations in furnace pressure have negligible effects on fuel flow rates. As a result of constant fuel flow rates mixing with varying air flow rates, the air-fuel ratio leaving the burner is also varying.

The effect, in turn, of varying air flow rates and air-fuel ratios entering the furnace is in two parts. The total volume flow rate perturbations begin immediately upon leaving the burner to mix with the gases in the furnace, decelerating and generating acoustic waves which propagate away in all six directions. Combustion takes place at some later time (the combustion time delay), farther out in the furnace. Heat release rates vary only as a function of the varying air-fuel ratio. When the burner is operating very fuel-rich (a combustion modification for the purpose of NO_x control) the heat release rate varies strongly with air-fuel ratio. The varying heat release rates also create acoustic waves which propagate away in all six directions.

After appropriate time delays for acoustic wave travel to the limits of the furnace cavity, and reflection off solid boundaries (at some efficiency) the waves return (at different times) to the burner exit and add together to create the furnace pressure variations which, in turn, cause further variations in air flow rates and air-fuel ratios coming from the burner. Acoustic waves which travel to the exit of the radiant section of the boiler cause varying rates of flow of gases out of the radiant section into the back pass.

The resulting analytical model resolved, in the limit, to the conventional rocket engine model (a verification of the basic model). It was also verified reasonably well by comparison with the available data. The analytically predicted instability frequencies of 11 and 43-45 Hz agreed well with measured frequencies of about 12.5 and 40-50 Hz. The model properly predicted a strong instability at 11 Hz. Although the frequencies in the 40-50 Hz range were properly predicted, the model did not clearly predict an instability in this frequency range. The model also properly showed that the vertical distribution of burners in the burner array prevented a possible instability in the intermediate frequency range, at about 30 Hz. In general, the agreement with data is considered good for this kind of analysis.

Further study of the characteristics of air-side feed system coupled modes of instability such as this in utility boilers showed two significant results relative to combustion modifications made for the purpose of NO_x control. If the burners are operated very fuel-rich, near to the rich flammable limit, the variation of the heat release rate resulting from air-fuel ratio perturbations is very large (the gain in the loop is high) and the boiler tends to become unstable. Boiler vibrations were consistently shown to become significant when more than about 25 percent of the burners were operated air-only, for purposes of NO_x control. It was shown that the burner air-fuel ratio, under such operation, approaches the natural gas fuel-rich flammable limit. Unless care is taken to reduce the loop gain, combustion instability could become a very real, practical limit to NO_x reduction.

The second observation relates to the anchoring of the flame in the burner. If the fraction of combustion which is completed within the burner is a function of the flow velocity in the burner, the flow rate through the burner becomes more sensitive to furnace pressure perturbations and combustion vibrations are amplified. All of the cases of high vibration observed in this data sample from natural gas-fired boilers (and one, undocumented case with oil fuels) involved relatively poorly anchored flames in the burner. Aerodynamic (vortex) flame stabilization appears particularly sensitive to variations in flow velocity through the burner. Again, fuel-rich burner operation creates a more difficult flame anchoring problem and, in turn, a greater possibility of unstable combustion. Unless recognized and properly corrected, this could also represent a limit on NO_x reduction.

In general, however, the analysis of air-side feed system coupled modes of instability in utility boilers shows that, while modifications to the combustion in existing boilers can create unstable combustion and thus limit NO_x reduction, such instabilities can be prevented by analysis and proper design. The worst case, in designing for stable combustion with burners operating near the rich flammable limit, must be resolved for other reasons. Simply to maintain a flame in the furnace the burners would have to be designed with somewhat less fuel-rich flame ignition and stabilization zones, with subsequent controlled mixing and flame propagation into the more fuel-rich regions. These design changes are simultaneously those which minimize combustion instability. Therefore, there is no reason to conclude that combustion instability represents an inherent limit to NO_x reduction.

SECTION IV

PLANT EFFICIENCY

During the study described by Dykema (1), data were accumulated on electrical load (the useful plant output) and fuel flow rates (heat input) for most of the test conditions involving natural gas and oil fuels. These are the two parameters of real, practical interest from the standpoint of energy efficiency; the electrical power which can be generated for a given heat input from the fossil fuel. In the overall study, of which this task is a part, the interest has been, and is, in the control of NO_x emissions. Energy efficiency, then, represents a constraint on NO_x reduction techniques in that implementation of a NO_x reduction technique, by definition, shall not significantly reduce the energy efficiency of a plant.

The initial impetus for the efficiency study derived from the observation that the overall plant energy efficiency varied over a range of as much as 12 percent. A simple, direct computer correlation was attempted at that time, relating efficiency directly to the measured NO_x data, using a second-order polynomial. The resulting correlation coefficients were moderately low (0.34 and 0.58 with oil and natural gas fuels, respectively). Those results tended to indicate that techniques used to reduce NO_x had no significant effect on plant efficiency. The correlation coefficients were sufficiently high, however, to suggest further investigation. If there were a direct relation, the derived correlation equations indicated that if the NO_x levels were reduced from the uncontrolled levels (440 ppm for oil and 660 ppm for gas) to 100 ppm, reductions in efficiency of 7.0 and 9.6 percent, respectively, could result. Such efficiency losses would indeed be considered significant. If such losses were shown to result from the modifications made for the purpose of NO_x reductions, then that technique for NO_x control could not be considered acceptable.

The objective of this task, then, was to identify whether (and how much) combustion modifications made for the purpose of NO_x control might affect the energy efficiency of a utility boiler. If the dimensionless ratio of electrical output (load) to heat input from the fuel is considered an acceptable measure of the overall energy efficiency of a utility boiler, then a large sample of appropriate data was available (1). In using that data for this study it was assumed that if there were no need to control NO_x emissions the boilers would be fired such that: (a) all burners were active, (b) no NO_x ports were used, (c) overall boiler excess air would be only slightly higher than that necessary to assure acceptably low carbon monoxide (CO) emissions, and (d) boiler load would be varied as necessary to meet electrical output requirements. Tests involving burners out of service (air-only) and air flow through NO_x ports, then, represent tests of combustion modifications (from the normal mode) made for the purpose of NO_x reduction.

4.1 COMBUSTION MODIFICATIONS FOR NO_x CONTROL

It seems clear that the formation of the rather trace quantities (parts per million) of NO_x in a boiler cannot be the direct cause of any measurable changes in overall plant efficiency. Instead, NO_x emissions vary in response to variations in some other parameter. The variations in that parameter, then, may also cause the plant energy efficiency to change. There are a number of operational and geometric parameters in boilers variations of which can have significant effects on NO_x emissions. Some of these parameters must be varied in normal operation and in design simply to satisfy the main purpose of a utility boiler; the demand for electrical power. Some of these parameters, however, can be deliberately varied for the sole purpose of reducing NO_x emissions. It is the simultaneous effect of variations in these latter parameters on plant efficiency that is of interest here. In order to use all of the data available it was necessary to correct for, or weed out, the effects on plant efficiency of variations not for the purpose of NO_x control so that the effects of those that are made for the purpose of NO_x control can be seen and evaluated.

For example, we know that strong reduction in boiler load will, in almost all cases, strongly reduce NO_x emissions, with a simultaneous significant effect on plant efficiency. Although some load sharing with other boilers can take advantage of this observation to minimize overall NO_x, normally the load on a given boiler must be varied to accommodate the smaller variations in electrical demand regardless of the effect on NO_x emissions. Load variation, then, is not considered a NO_x control technique in this study. The installation of NO_x ports, however, is a geometric and operational variable made for the primary purpose of NO_x control. It is important in this study to try to evaluate, from experimental data, the simultaneous effect of NO_x ports on plant efficiency.

Similarly, the study (1) showed that, at least with natural gas and oil fuels, the effect of overall boiler excess air on NO_x emissions was small. In any case, all indications are that the plant efficiency is maximum with minimum excess air, compatible with acceptable emissions of CO and smoke. As a result, control of excess air, at least for this study (involving natural gas and oil fuels), is not considered a significant NO_x control technique.

Reduction in combustion air temperature is known to be an effective NO_x control technique if the NO_x emissions are largely thermally generated. Although combustion air temperatures varied somewhat in the data (1), they varied only as a result of, and in conjunction with, boiler load (or total fuel and air flow). There are no data in Ref. 1 directly showing the effect on NO_x emissions of combustion air temperature alone. Some efforts were made (1) to analytically separate this effect. Because this modification can have a significant effect on NO_x emissions, and might be deliberately varied to control NO_x, a similar effort is made here to evaluate the magnitude of the effects of combustion-air temperature reduction on plant efficiency.

Although load variation is not considered here as a NO_x control technique, it does have a significant effect on efficiency. In order to use all available data to determine the effects of NO_x control techniques, it was necessary to account for, or correct for, efficiency variations resulting from load variations, as well as from all other hardware and operating condition variations which are not specifically for the purpose of NO_x control.

4.2 EFFICIENCY LOSSES

Six areas of efficiency losses were considered in this study. Three considered heat losses from the combustion gases (heat not transferred to the steam) and two considered efficiency losses in the steam and electrical processes. The final area was an "other" category. Of these six areas. four are reasonably well-known and can be calculated: electrical generating and control equipment; uncondensed water vapor in the flue gases; excess sensible heat entering the stack in the flue gases; and incomplete combustion. Data were not available with which to calculate variations in the energy expansion efficiency through the steam turbine. A reasonable functional form of these losses could be assumed however and quantified from the data. After subtracting these five losses from the overall plant efficiency, the remaining category could still show some effects of combustion modifications which were not accounted for in the calculated losses.

Obviously, efficiency losses in the electric generating and control equipment, at a given electrical load, are independent of how the heat is being supplied to the steam in the boiler. Thus, these losses could not be a function of combustion modifications made for the purpose of NO_x control. This loss is calculated, and the efficiency corrected, only to increase the sensitivity of the corrected efficiency data to other variations.

It appears desirable to maintain the temperature of the flue gases entering the stack well above the dew point of the water vapor to avoid the corrosion which can result from condensation of water containing dilute acids on cooler surfaces (such as the air pre-heater). As a result, the heat losses due to uncondensed water vapor in flue gases are unavoidable and will exist regardless of any combustion modifications. Actually this loss appears in the data of this study only because the initial efficiency calculation, following convention, involved the high heat of combustion rather than the low heat. Again, this loss is calculated only to increase the sensitivity of the corrected efficiency to other variations.

One way in which combustion modifications could reduce plant efficiency would be by reducing the heat transferred to the steam cycle and increasing the sensible heat losses in the hot flue gases entering the stack. These increased heat losses could occur both through increased heat capacity (specific heat) and the temperature of the flue gases. Since the heat capacity of the flue gas is largely determined by the fraction of inert nitrogen in the air, it is almost totally a function of the overall boiler air-fuel ratio, or the excess air. Small changes in the composition of the combustion products have a negligible effect on the heat capacity. While it must be accounted for in calculating sensible heat losses, it was calculated in this study simply as a function of the overall boiler air-fuel ratio and, as a result, was not a function of any combustion modifications.

On the other hand, increased losses due to combustion modifications could be reflected in higher flue gas temperatures at the same boiler air-fuel ratio. Unfortunately, the temperature of the flue gases leaving the pre-heater (and entering the stack) was not recorded in many of the tests available to this study. Review of such temperature data as were available indicated that they could fit reasonably well as a linear function of the plant electrical load. Combining this temperature function with the heat capacity function discussed above, the efficiency losses for all of the data were calculated and examined. This showed that, for a given boiler, efficiency losses (not heat losses) from this source ranged from about 1.9 percent at minimum load to about 3.1 percent at rated load. This range largely resulted from flue gas temperature variations of about 39 K (70°F). These flue gas temperature variations are clearly reflected in the combustion air temperatures, physically through the pre-heater. Flue gas temperature variations of about 16 K (30°F) or higher, corresponding to one-half percent or more efficiency losses, are easily detectable through combustion air temperature variations. The latter measurements were available to this study for most data. Further review of combustion air temperature data showed no consistent variations greater than 16 K (30°F) as a result of combustion modifications (at any given load and level of excess air). Therefore, it was assumed that any effects of combustion modifications on sensible heat losses must amount to less than about one-half percent efficiency variation. The efficiencies calculated from measured data were only accurate to within 1 to 2 percent. As a result, further effort to detect any efficiency variations due to sensible heat losses was terminated. As with the two previous efficiency losses, however, the overall efficiency was corrected for these sensible heat losses, again to make the remaining efficiency variations more sensitive to other effects.

The area of efficiency loss which appears the most likely to be significantly affected by combustion modifications is that identified here as incomplete combustion. The most effective general technique for NO_x control demonstrated in the data is the so-called two-stage combustion. In this approach, the first stage is operated fuel-rich, and appreciable cooling of the products of this fuel-rich combustion is accomplished before the remaining air is introduced to provide fuel-lean combustion in the second stage. One can easily visualize that if the first stage were operated too fuel-rich the flame might actually be extinguished (and the efficiency drop to zero). Also, the products of fuel-rich combustion still contain considerable chemical energy (for example, in high concentrations of carbon monoxide). If these products are cooled too much before the remaining excess air is added, further reaction to minimum energy products (for example CO to CO_2) could be inhibited, with a resulting large loss in efficiency. Thus, efficiency losses due to incomplete combustion were of most interest in evaluating the effects of combustion modifications.

The final major loss which clearly affects the efficiency data variations is that due to steam expansion losses through the turbine. These losses could be calculated directly if the flow rates and thermodynamic properties of the steam at the turbine control valves and at all exits from the turbine system were known. Unfortunately, these data were not gathered in the study reported in Ref. 1. It is known, however, that the need to vary the power output of the constant speed turbine to match the desired electric generating load results in operation at less than maximum turbine efficiency over most of the load range. Losses resulting from this mismatch are clearly independent of any combustion modifications.

It is possible that changes in the heat release and heat transfer profiles through the boiler, besides changing the exit flue gas temperature (total heat transferred), could also change the overall thermodynamic cycle efficiency. For example, staged combustion could reduce the combustion temperature and the heat transferred in the radiant section of the boiler while raising both in the area of the superheater tubes. In such a case it might be possible that the resulting average temperature difference (gas-to-water or steam) might be lower and the resulting steam cycle efficiency higher.

The large variations in efficiency losses with load through the steam turbine were at least partially accounted for in this study by assuming that these losses would follow some parabolic function of load, with the maximum efficiency probably occurring (by design) at the most common load, about 80 percent of rated load. A single expression for this loss, as a function of load alone, was empirically determined for all data on a given boiler (turbine design), regardless of the fuel burned or the combustion modification in use. These losses were then subtracted from the overall efficiency data. Any effects of combustion modifications on thermodynamic cycle efficiency, then, would be more clearly observed in the remaining efficiency data.

In this discussion, it has been implied that the effect of a given combustion modification on efficiency could be seen directly in the difference in measured efficiency with and without the modification. It must be remembered, however, that the utility boilers involved in the tests were also generating electricity for commercial use during the tests. It is normal practice for the boiler operator, not always intimately involved in the environmental testing in progress, to make certain operational adjustments to maintain boiler operation as close to optimum as possible. Thus if a given modification is made to the combustion to evaluate its potential to control NO_x, and the change results in a severe decrease in plant efficiency, the boiler operator (or even some automatic control equipment) may adjust operating conditions to recover all or part of this loss. For example, if a given configuration of active burners and burners out of service resulted in higher temperature flue gases entering the stack, the boiler operator might increase the rotational speed of the air preheater to transfer more heat to the incoming air and recover this loss. Wherever possible, then, the modified and nonmodified (normal) operating conditions need also to be examined to determine if operating conditions are significantly different. Since there were not sufficiently detailed data in Ref. 1 to evaluate all possible operational changes, a conclusion that a given combustion modification did not significantly affect efficiency must be considered to imply also that the efficiency was at least not affected beyond the ability of the operator to restore operation to normal by changing some other part of the system.

4.3 RESULTS

Evaluation of the effect on plant efficiency of combustion modifications made for the purpose of NO_x control was conducted in three steps:

a. The overall plant efficiency data were corrected to remove the influences of the four efficiency losses which could be calculated directly. The calculated losses due to incomplete combustion were then evaluated to determine if combustion modifications had any significant effect. The other three areas of efficiency losses were not evaluated further.

b. An empirical relation was established, from the data, to account for efficiency variations due to variations in expansion efficiency through the steam turbine. Although this loss cannot be charged to combustion modifications made for the purpose of NO_x control, the simultaneous effects of load on efficiency and NO_x were evaluated.

c. Finally, the plant efficiency data, already corrected for the losses in a., were further corrected for the loss variations from b. and the resulting data evaluated for any other effects of combustion modification.

Results described herein do not concern losses in the electrical generating and control equipment or those due to uncondensed water vapor or excess sensible heat in the flue gases. These latter corrections are described briefly below.

4.3.1 Efficiency Corrections

As described in Appendix A, a simple three percent load loss was assumed in the electrical generating and control equipment for all plants and for all loads. This percentage was somewhat arbitrarily chosen as a typical loss for such systems. The main observation, however, is that this loss is relatively small and variations from this nominal level should be negligible.

Heat losses due to uncondensed water vapor in the flue gases are a constant fraction of the heating value of the fuel for the natural gas and low sulfur oil fuels used in the test data (1). These heat losses represent 9.63 and 5.60 percent of the higher heating value of these fuels, respectively. The overall plant efficiency, following convention, used the higher heating value. In correcting the plant efficiency data for the uncondensed water vapor, a loss certainly not chargeable to combustion modifications, we are, in effect, using the low heat of combustion. As in all of these corrections the efficiency correction does not amount to the full 9.63 or 5.60 percent heat loss, because more than half of this heat would be lost due to the low steam cycle efficiency. For a typical gas-fired boiler the efficiency correction for uncondensed water vapor amounted to increasing the remaining efficiencies by 4.1 to 5.2 percent.

The sensible heat losses are not a constant fraction of the heating value of the fuel but also depend on the air-fuel ratio at which the fuel is being burned. These losses are greater at high air-fuel ratios, or levels of excess air, because more total flue gas leaves the boiler at any given temperature. This is somewhat offset by a decrease in flue gas heat capacity with increasing air-fuel ratio but the overall effect is still for these losses to increase with air-fuel ratio, or excess air.

For all of the data of Ref. 1 from one facility (six boilers) firing natural gas and oil fuels, the efficiency corrections for sensible heat losses ranged from 1.6 to 3.1 percent. The data from the other facility (two boilers) showed corrections in the range of 2.5 to 3.8 percent. Since these calculated corrections involved functions only of boiler load and overall boiler excess air they could not be expected to show any effects of combustion modifications, such as burners out of service, so no significant attempt was made to further analyze the calculated corrections. One clear result is the conclusion that reduction in excess air increases efficiency. The NO_x analysis of Ref. 1 has already indicated that, at least with those utility boiler data, reduction in excess air slightly increases NO_x with natural gas and slightly decreases NO_x with oil fuels. In neither case are the efficiency or NO_x variations large. Coal-fired boilers, however, should show the same trend of efficiency with excess air but decreasing excess air can significantly reduce NO_x. In general, then, decreasing excess air increases the plant efficiency and generally decreases NO_x emissions. As discussed earlier, however, reductions in excess air are not really considered a combustion modification made primarily for the purpose of NO_x control.

4.3.2 Incomplete Combustion

An area of efficiency losses which can be reasonably calculated and can be significantly affected by combustion modifications involves incomplete combustion. As shown in Appendix A, these losses can be estimated from flue gas measurements of the carbon monoxide, carbon dioxide and oxygen. These data can then be evaluated to determine, directly, if combustion modifications significantly affect this loss.

As discussed in Appendix A, the measure of the completeness of combustion used in this study is essentially a carbon loss calculation. In the few tests where unburned hydrocarbons were measured, they appeared to be negligible. In all cases CO was negligible, at least as a source of significant energy loss. The assumption was made, therefore, that all carbon from the fuel which does not appear as CO_2 in the flue gas analyses is lost (does not yield any heat). Complete combustion (zero loss) is defined as the case where the measured CO_2 is equal to that calculated from stoichiometry corresponding to the measured O_2.

Figure 16, in Appendix A, already shows that the combustion modifications, such as result from the use of burners out of service or NO_x port techniques, have no discernible effect on losses due to incomplete combustion. The measured levels of CO_2 consistently appear slightly less than one-half percent below those calculated from stoichiometry, at any measured level of O_2. The significant observation is that measured CO_2 data from boilers fired with all burners active and NO_x port closed (nominal design condition) are the same, within the scatter of the data, as those where the combustion was modified (for NO_x control) with NO_x ports open, one-third of the burners operated air-only or both.

The O_2 and CO_2 data were used to calculate a correction to the overall plant efficiency, to remove the influence of this loss from the overall data so that the remaining data could be analyzed further. These calculated corrections were also reviewed to again evaluate whether there are any significant differences between losses due to incomplete combustion when the boilers were operated in the nominal condition and when these same boilers were operated with modified combustion. The data examined here were the differences between the average calculated efficiency corrections for this loss under these different operating conditions.

The resulting data are shown in Table 1. Of the sixteen possible combinations of boiler types, burner configurations, NO_x port operation and fuels, the effects of combustion modifications on the efficiency correction for incomplete combustion were well within the accuracy of the data (data scatter) in thirteen of the combinations, ranging from an increased loss of 0.6 percent to a decrease in this loss of 0.8 percent. No data were available from two combinations and one showed an increased loss of 2.4 percent. This latter case, from the smallest, single-wall boiler, apparently resulted from the fact that only four tests were available of the nominal configuration and these involved CO_2 measurements made by a different company, using different apparatus, than those made under the modified configurations (or on any other boilers or with the gas fuel). As a result the 2.4 percent loss is not considered valid. It certainly is not consistent with all of the other thirteen observations.

This evaluation of the effects of combustion modifications on efficiency corrections agrees with the previous conclusion directly from the CO_2/O_2 data. This conclusion, and the conclusion of this study, is that combustion modifications for the purpose of NO_x control do not significantly affect efficiency losses due to incomplete combustion.

4.3.3 Effects of Load Variations

As discussed several times to this point, the variation of load is considered a necessary operational function of an electric generating plant and, as such, is not considered a combustion modification made for the purpose of NO_x control. It is well-known that a reduction in boiler load normally results in a reduction in NO_x concentrations measured in the flue gas (although not necessarily in NO_x emissions expressed as grams per unit heat input). Although the necessary data were not in the sample available to the previous study (1), it was suggested here that the reduction in measured NO_x concentrations with load might be due to the simultaneous observed reduction in combustion air temperatures with load. Reduced combustion air temperatures reduce initial combustion product temperatures which, in turn, reduce the rate of NO_x formation by the thermal mechanism. Control of combustion air temperatures is a valid combustion modification which could be made for the purpose of NO_x control.

Unfortunately, in the data sample used (1), reductions in boiler load were always accompanied by reductions in combustion air temperatures. Since many other changes take place when the load is changed, there was no suitable, valid way in that study (1) to evaluate the temperature effect alone. Similarly, there is no good, valid way in this study to investigate the effect of combustion air temperature alone on plant efficiency. It is again necessary, therefore, to look only at the combined effects of all parameters which vary with load and it is still necessary to define that load variation is not a

TABLE 1. EFFECTS OF COMBUSTION MODIFICATIONS ON PLANT EFFICIENCY LOSSES DUE TO INCOMPLETE COMBUSTION

Boiler No.	Firing Type	NO$_x$ Parts	Rated Load, MW	No. Burners	Difference in Efficiency Corrections, %*							
					Firing Configurations**							
					Natural Gas				Oil			
					A*	B	C	D	A	B	C	D
1	Single-wall	No	180	16	-	-	-0.6	-	-	-	+2.4	-
2			240	12	-	-0.3	-	+0.1	-	+0.6	-	-0.3
3	Opposed	Yes	240	12	+0.6	+0.1	-	-0.1	+0.4	N.A.	-	N.A.
4			360	24	-0.5	-	-	-0.1	+0.2	-	-	-0.8
				Averages	+0.1	-0.1	-0.6	0	+0.3	+0.6	+2.4	-0.6
				Averages	Both Gas and Oil				+0.2	+0.2	+0.9	-0.3

Firing** Config.	Fraction Burners Air		NO$_x$ Ports	
	Nominal	Modified	Nominal	Modified
A	None	None	Closed	Open
B	None	1/6	Closed	Open and Closed
C	None	1/4	Closed	
D	None	1/3	Closed	

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* Difference in Efficiency Correction = The correction for losses due to incomplete combustion in the modified operating configuration minus the correction in the nominal configuration.

combustion modification made for the purpose of NO_x reduction. The attempt here is only to develop a method of correcting for the major effects of efficiency which are not affected by combustion modifications.

For this purpose it was assumed that the expansion efficiency through the steam turbine should be the major effect of load on plant efficiency. It was further assumed, subject to experimental verification, that this efficiency variation should be described by some parabolic curve (second-order polynomial) of efficiency as a function of load. It was expected that the maximum would occur near the most common operating point (about 80 percent of rated load). The appropriate curve should be the same for a given boiler type but independent of fuel type (natural gas or oil) and certainly independent of combustion modifications resulting from the use of burners out of service or NO_x ports. The actual magnitude of the efficiency, of course, could vary as a function of many other variables, but any remaining magnitude variations could be evaluated in the final corrected efficiencies data.

The data used to develop this load correlation were the original plant efficiencies corrected for all four of the other calculated losses described in Sections 4.3.1 and 4.3.2. A number of data samples were correlated to evaluate differences in boilers, fuels and other operating conditions. The final correlation developed for the largest boiler in the sample is shown in Figure 1. This same correlation curve, reduced by a constant three percent, also correlated all of the data from the rest of the boilers in the sample. The correlation coefficient for all of the data, all boilers, both fuels and all operating conditions, was 0.744. This is considered an adequate correlation.

The efficiency curve shown in Figure 1 shows a maximum of 50.5 percent, at about 84 percent of rated load, and drops to as low as 44.3 percent at 40 percent of rated load, a decrease of more than six percent. Since load variations down to about 43 percent of rated load were part of the data, it is likely that most or all of the efficiency variations observed in the data, excluding data scatter, result from load variations. It follows, then, that little other efficiency variations remain which could be ascribed to combustion modifications made for the purpose of NO_x control.

The NO_x data from the same plant whose efficiency variations with load are shown in Figure 1, were correlated in the previous study (1). These correlations, for both fuels, are also shown in Figure 1. It can be seen that, while NO_x concentrations from both fuels decrease with load (as does the efficiency), neither follows the parabolic shape of the efficiency curve. If anything can be interpreted from this comparison, it would appear that the NO_x concentrations probably follow the combustion air temperature variations and the efficiency follows the steam turbine performance, both of which are, in turn, associated with load variations. Without further data however these separate, and perhaps little related effects cannot be verified.

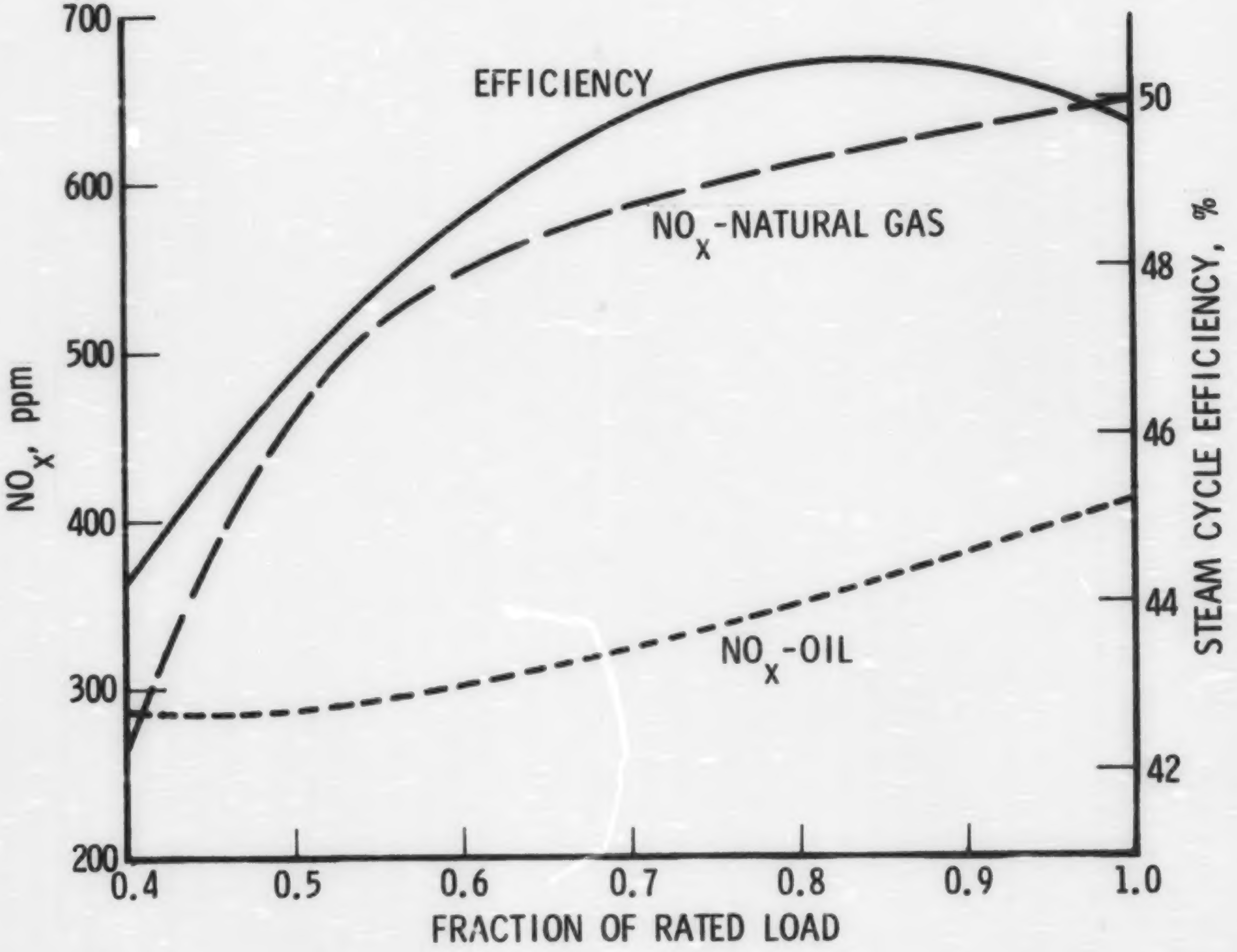


Figure 1. Effects of boiler load variation on efficiency and NO$_x$ emissions.

In any case, it is clear from Figure 1 that, at least for these boilers, load reduction does indeed reduce NO_x concentrations in the flue gases but it also substantially reduces plant efficiency. The efficiency reductions shown correspond to the majority of the efficiency variations observed in the data. There is not sufficient data in the available sample to determine whether the two effects are separate and, therefore, whether this same NO_x reduction could be attained without effecting the efficiency.

The correlation of corrected efficiency versus load discussed here was finally used to further correct the efficiency for a final evaluation. That final evaluation is discussed in Section 4.3.4.

4.3.4 Other Possible Effects

After correcting the original plant efficiency data for the four losses discussed in Sections 4.3.1 and 4.3.2 and for the load effects discussed in Section 4.3.3, very little variation remained outside of the expected data scatter. Estimated precision of the original (instantaneous and simultaneous) measurements of fuel flow and electrical load, and of the various other measurements which were used in efficiency corrections, resulted in the conclusion that the precision of the final, corrected efficiency values could not be better than about two percent. This was about the range of what appeared to be the data scatter in the final efficiency data. Nevertheless, some effort was made to evaluate any other possible effects of operation with burners out of service, with NO_x ports, or both.

Several attempts were made to evaluate possible mechanisms by which these combustion modifications might affect efficiency. The one, if any, which appeared to have a discernible effect was based on the assumption that cooler combustion in the radiant section of the boiler, resulting from fuel-rich first stage operation (in turn resulting from burner and NO_x port variations) might cause an efficiency loss. An expression was developed which related the average combustion temperature in the active burner region of the boiler when the active burners were operated fuel-rich to that same temperature under nominal boiler operating conditions. Reasonable data correlation coefficients were obtained using this term but the maximum possible efficiency loss with staged combustion was less than one percent. Considering the estimated range of data scatter, and this very small potential efficiency loss, it was concluded that no further significant effects of combustion modifications, made for the purpose of NO_x control, could be observed in the data.

After correcting the calculated efficiency data for all of the losses discussed in this report (including the one of this section) the remaining data had a standard deviation of 2.2 percent.

SECTION V

COMBUSTION STABILITY

During the previous study (1) data available from a number of tests in natural gas-fired utility boilers were examined to determine if a general mechanism could be identified which might account for cases of observed combustion instability, flame liftoff or both. Results of that examination led to the tentative conclusions that; (a) the combustion instability was very likely an air-feed system coupled mode, and (b) the feedback coupling between air flow velocity through a burner and the degree of combustion within the burner has a strong effect not only on the steady-state air-fuel ratio within the burner, but also on both combustion instability and flame liftoff. The experimental observations on which these conclusions are based are discussed in the earlier study (1). It was not within the scope of that study, however, to do more than evaluate potential mechanisms from study of available data. Development of an analytical model of the suspected instability mechanism and verification of analytical predictions against this data, were deferred to the subject study. This section reports these analytical efforts.

5.1 ANALYTICAL MODELING

The analysis is divided into three components; (a) response to air flow through a burner to perturbations in furnace pressure at the burner exit, (b) effects of perturbations in total flow and air-fuel ratio on furnace pressure, and (c) coupling of the above two responses into a complete description of the combustion-air feed system coupled mode of instability. Details of the entire analysis are contained in Appendix B.

5.1.1 Burner Air Flow Response

Figure 2 shows a schematic of the model of air flow through a burner which was used as the basis for this part of the analysis. Major assumptions include:

a. The windbox dimensions are small compared to those of the furnace cavity. This results in the further assumption that the air pressure in the windbox is uniform throughout the windbox.

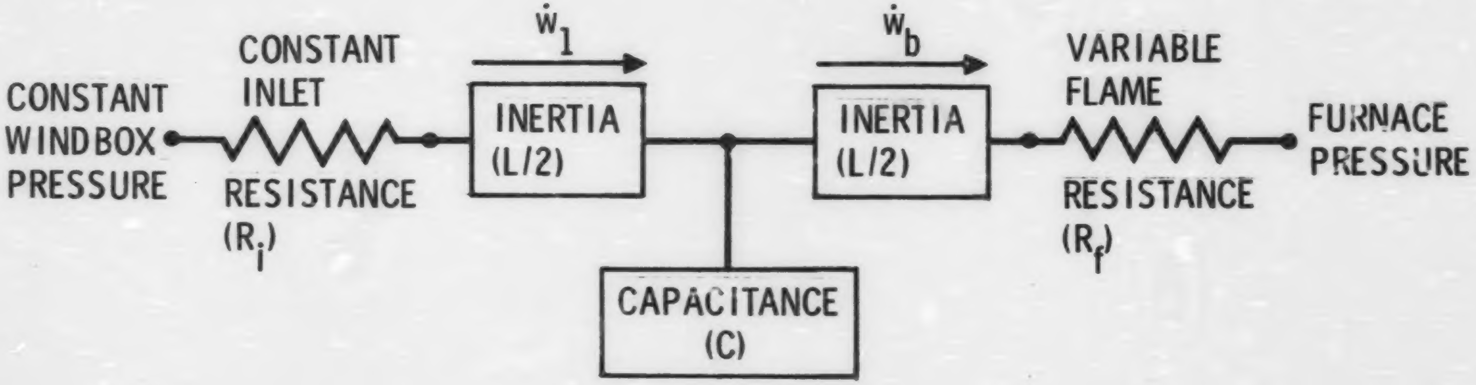

CONSTANT
WINDBOX
PRESSURE
CONSTANT
INLET
RESISTANCE
(R_i)
$\dot{w}_1$
INERTIA
(L/2)
CAPACITANCE
(C)
$\dot{w}_b$
INERTIA
(L/2)
VARIABLE
FLAME
RESISTANCE
(R_f)
FURNACE
PRESSURE

Figure 2. Model of the dynamics of a burner in a utility boiler.

Reproduced from
best available copy.

b. The windbox volume is large compared to the volume flow of air in and out of the windbox, at least over the period of one cycle over the frequency range of about 10 to 100 hertz. This together with the previous assumption, results in the further assumption that the windbox is a true plenum and its pressure is constant both in time and space.

c. The resistance to air flow through the burner is concentrated in only two places; the inlet loss through the air registers and the exit loss resulting from the requirements for increased momentum due to the presence of a flame within the burner.

d. The axial length of the burners is small compared to the wavelengths of the oscillations of interest (up to 100 hertz). As a result, the burner air can be treated as two simple incompressible lumps with all of the burner capacitance concentrated between.

Assumption a. is certainly true in the horizontal direction parallel to the burner flow axes since the windbox depth is normally less than one-quarter of the furnace depth. Similarly, in the vertical direction, the windbox height is normally 30-45 percent of the furnace height. In the horizontal direction perpendicular to the burner flow axes (width), the windbox width may be quite comparable to the furnace width and assumption a. may not be valid. The windbox, however, is actually cluttered with burners, gas and oil feed lines, register control arms and linkages and other control and instrumentation lines. As a result, transverse flow and/or resonances within the windbox would be strongly damped.

In order to assure, with reasonable confidence, that all burners are equally fed air from the common windbox, the windbox flow area (and, therefore, volume) must be large relative to the sum of the burner flow areas. While some pressure measurements taken over a single windbox indicate a marginal flow area in the windbox, it is probably sufficient to justify assumption b.

Most burners are designed with air registers which are intended to impart some degree of swirl to the entering air flow. To generate this tangential flow component, some pressure head must be converted to velocity head. This is the definition of flow resistance. In addition, the air-only flow cross-section within the burner is again cluttered with fuel feed lines, diffusers and ignitors, while the flow cross-section after the fuel is introduced is relatively free of flow obstruction. It is reasonable to lump these pressure losses within the burner air-only flow region into a single inlet resistance, as described in assumption c. Also, if there is a significant flame within the burner, it would tend to be located primarily near the burner exit, within a short burner length, where the resistance to unheated air-flow alone would be small. This part of assumption c., then, also seems reasonable.

Assumption d. is simply a normal method of modeling compressible flow in a pipe for purposes of dynamic analysis. Although not assumed beforehand, it may even be adequate to model the air within the burner as a single incompressible lump, at least for analyses within the frequency range of interest.

Details of the development of the analytical expression for burner response are contained in Appendix B. Major results are discussed in Section 5.2.

5.1.2 Furnace Pressure

In Section 5.1.1, the variations in air flow through a burner in response to perturbations in furnace pressure were described. As this varying air flow (now with a constant fuel flow mixing into it) enters the furnace it causes various three-dimensional perturbations in flow within the furnace cavity.

Initially, any excess in flow velocity will be quickly damped by momentum exchange with the relatively stagnant gases already in the furnace. This deceleration can result in generation of acoustic waves which would then propagate away from the burner exit in all directions at the local speed of sound.

Combustion may be occurring simultaneously with this flow deceleration. The mixing required for flow deceleration largely occurs at the periphery of the main flow coming out of the burner (core flow) while that required to complete combustion may be either within the core flow (combustion at the air-fuel ratio of the burner) or at the periphery (combustion at the furnace air-fuel ratio), or both. As a result, the average time delay to complete combustion in most cases will be longer than, or at least equal to, that required to damp the unreacted flow perturbations. These relative delays will depend on the burner design, the fuel state and the difference between the burner air-fuel ratio and that represented by the furnace gases at the burner exit.

It seems clear, however, that the time delay to damp the unreacted burner flow perturbations is short compared to the period of the very low frequency oscillations of interest in full-scale utility boilers. Therefore, it is reasonable to set the time delay for damping of burner flow oscillations to zero and to assign a variable time delay (τ_c) to represent the average time required for complete combustion.

The effect of combustion is to change the local gas density both by raising the temperature and by changing the molecular weight between the reactants and the products. These changes can also generate three-dimensional acoustic waves which again propagate away at the local speed of sound. Thus, the effect of a single perturbation in flow out of a burner can be to generate acoustic waves both instantaneously at the burner exit (flow perturbation damping) and (usually) at some later time and spacial location further out in the furnace (combustion). This model of dynamic burner flow behavior, is shown schematically in Figure 3.

As discussed thus far, the effects of burner flow-perturbations in generating furnace pressure perturbations are complicated, with two sources of acoustic perturbations occurring at different times and spacial locations, and three-dimensional acoustic response of the furnace cavity. Consideration of complete three-dimensional wave equation solutions for the furnace cavity, with driving at two (variable) sources would greatly complicate analysis of the overall coupled instability loop and very likely force a nonlinear computer solution. In the light of the approximations and assumptions discussed in Section 5.1.1, and other poorly known inputs, such as the appropriate average combustion time delay, a complete solution is not justified here. Perhaps such improvements could be incorporated at a later date if necessary to obtain adequate useful solutions. Several simplifying assumptions are made here, both to provide a means of coupling the dynamic, reacting burner flow to the acoustics of the furnace cavity as well as to develop manageable pseudo-acoustics for the three-dimensional furnace cavity. The major assumptions are discussed below.

a. Damping of burner flow perturbations is assumed to occur in a furnace cavity control volume at the burner exit. The time delay for this damping is short compared to the shortest period of oscillation of interest and is, therefore, taken as zero.

b. Although the burner mass flow perturbations are damped out immediately upon entering the furnace, the air-fuel ratio perturbations (resulting from varying burner air flow rates mixed with a constant fuel flow rate) remain and are carried farther out into the furnace cavity at a constant flow velocity. After an average time delay, τ_c, the air-fuel ratio perturbations result in perturbations in local gas temperatures and molecular weights during (concentrated) combustion. It is recognized that the combustion is actually distributed both in time and space and that the assumption of concentrated combustion is in disagreement with the observation discussed in Section 5.1.1 that some of the combustion has already been completed even before the burner flow enters the furnace

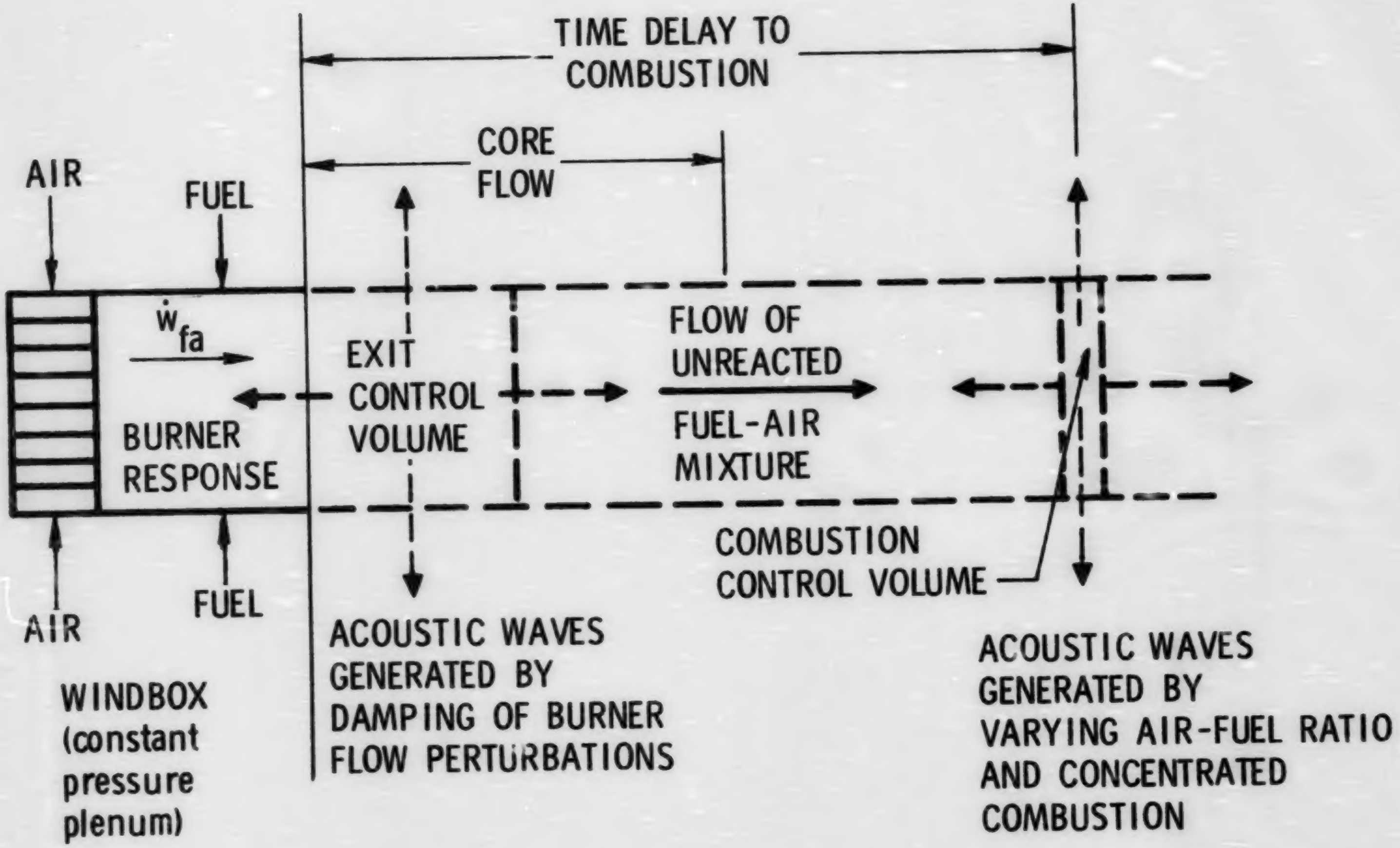

TIME DELAY TO COMBUSTION
CORE FLOW
AIR
FUEL
$\dot{w}_{fa}$
BURNER RESPONSE
EXIT CONTROL VOLUME
FLOW OF UNREACTED FUEL-AIR MIXTURE
COMBUSTION CONTROL VOLUME
AIR
FUEL
WINDBOX (constant pressure plenum)
ACOUSTIC WAVES GENERATED BY DAMPING OF BURNER FLOW PERTURBATIONS
ACOUSTIC WAVES GENERATED BY VARYING AIR-FUEL RATIO AND CONCENTRATED COMBUSTION

Figure 3. Active burner flow and combustion model.

cavity (partial combustion within the burner). This conflict does not introduce significant error, since Section 5.1.1 treats the beginning of combustion while this section concerns the average of complete combustion.

c. If the mass flow and combustion perturbations were introduced into an infinite reservoir, only small acoustic pressure perturbations would be generated. Instead, they would tend to produce large acoustic velocity perturbations. [This is because the specific acoustic impedance (the complex ratio of acoustic pressure to velocity) of the gases in the furnace cavity is low.] This observation is approximated here by the assumption that any changes in pressure which might tend to be generated by burner mass flow into, or combustion within, a control volume in the furnace cavity are exactly negated by a proportionate mass flow out of the control volume, in acoustic waves. This implies zero specific acoustic impedance.

d. In a finite enclosure, acoustic waves generated by burner mass flow perturbations or combustion, according to assumption c., can travel throughout the enclosure, reflect off solid surfaces and return to the source. The impedances at the surfaces of the control volumes, then, are no longer the specific impedance of the gas but depend on the location of the control volume with respect to solid surfaces, the damping of the acoustic waves in traveling through the gas and the efficiency of reflections off the solid surfaces. Pressures can now be generated within the control volume. This observation is approximated here by summing the flows into a control volume which result from returning, reflected acoustic waves. Pressures within the control volume result, through the perfect gas law, only from these acoustic flows.

e. It is assumed that the spherical, three-dimensional acoustic waves generated within a control volume can be resolved into independent plane waves traveling along the positive and negative Cartesian coordinates (six directions). With such pseudo-acoustics, allowance can also easily be made for acoustic damping in wave travel as well as for imperfect reflections off solid surfaces.

f. It is recognized that the plane waves are taken to travel from the control volume in which they were generated to a solid surface, reflect and return to the volume. For this analysis, the two control volumes described in assumptions a. and b. and their acoustic interactions are considered, but acoustic interactions between burners are not considered. The generated acoustic flow perturbations are divided equally among the six plane waves. Each plane wave requires a specific time delay to return to the source. The one exception to this is that wave which travels directly from the exit control volume back into the burner and into the windbox. A zero reflection is assumed for this wave.

Perhaps assumptions a. through f. should not really be called assumptions. Rather they are simplifying approximations of complicated, poorly known physical processes which are intimately involved in combustion stability. The separation of the total driving effects of perturbations in a gaseous, reacting flow into mass and reaction effects separated in time and space is an extension of an observation first suggested by Dykema (4) and developed further in applications to chemical laser combustors injecting gaseous propellants. The remaining assumptions, c. through f. are necessary to derive a simple set of driven, pseudo-acoustics for this three-dimensional case which are compatible with the state-of-the-art of understanding of these and other related phenomena.

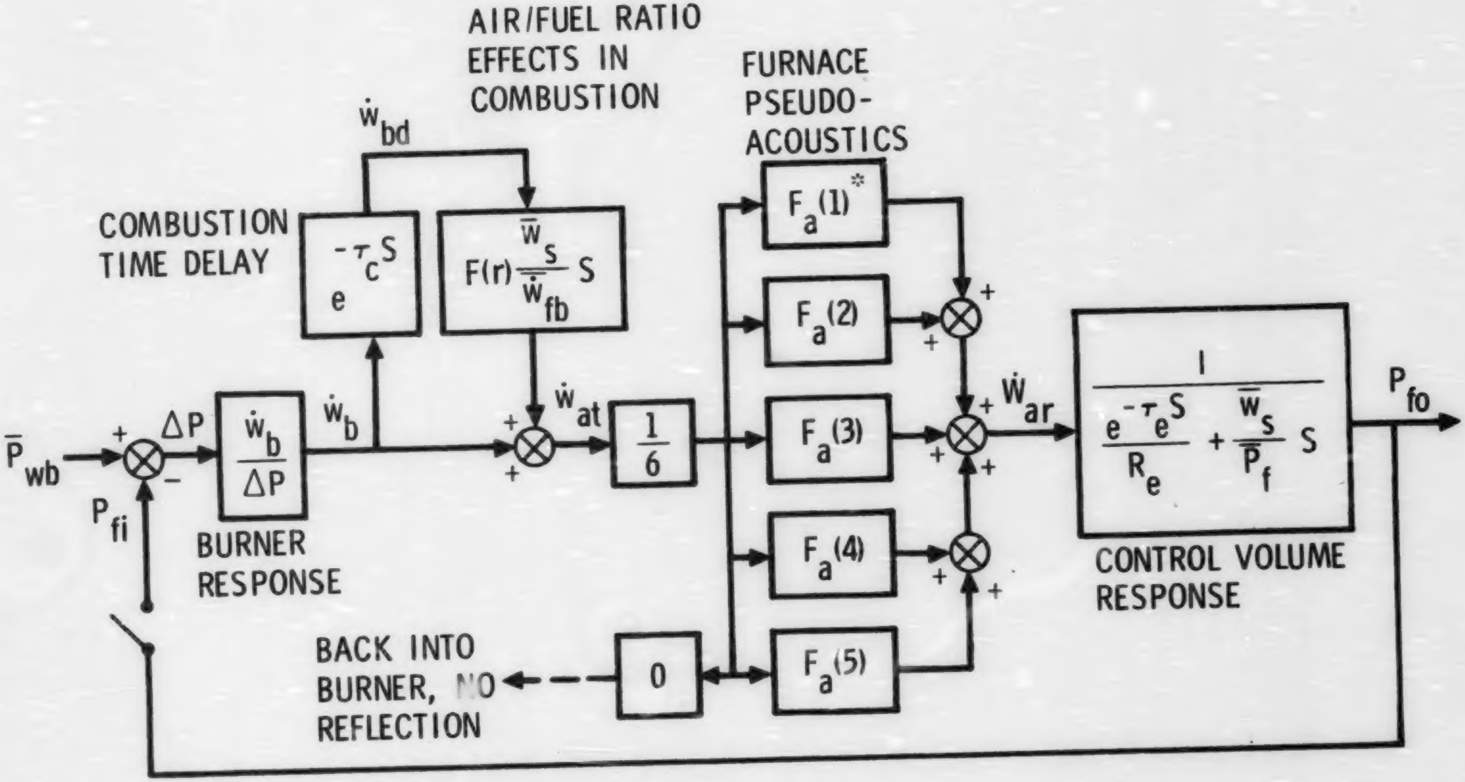
Based on the above assumptions and discussion, an expression for the response of furnace pressure (at the exit of a burner) to perturbations in the burner flow rate was derived. The details of that derivation are shown in Appendix B.

5.1.3 Feed System Coupled Instability

A block diagram of the response of the total, feed system coupled mode of instability is shown in Figure 4. The burner response shown in the figure is that response discussed in Section 5.1.1 and described by Eqs. (33) or (39) in Appendix B. The remainder of the forward loop shown in the figure represents the furnace response discussed in Section 5.1.2 and described by Eqs. (73) and (74) or (75) of Appendix B. The total, open loop response is simply the product of the burner response and the furnace response. Because the feedback is negative, the conditions for the closed loop to be unstable are that the phase angle between the output and input pressures (P_{fo}/P_{fi}) must be 180 degrees and the magnitude, or gain, of the open loop should be greater than one.

5.2 RESULTS

Results of interest here are of two types: (a) comparisons with other analyses and with data to evaluate the validity of the analysis, and (b) parametric studies of the effects of certain design variables on the analytically predicted stability of a boiler. The usefulness of the latter depends, of course, on the degree of agreement in the former.


AIR/FUEL RATIO EFFECTS IN COMBUSTION
FURNACE PSEUDO-ACOUSTICS
$\dot{w}_{bd}$
COMBUSTION TIME DELAY
$e^{-\tau_c S}$
$F(r)\frac{\bar{w}_s}{\bar{\dot{w}}_{fb}} S$
$F_a(1)^*$
$F_a(2)$
$F_a(3)$
$F_a(4)$
$F_a(5)$
$\bar{P}_{wb}$
+
−
ΔP
$\frac{\dot{w}_b}{\Delta P}$
$\dot{w}_b$
$\dot{w}_{at}$
$\frac{1}{6}$
$\dot{W}_{ar}$
$\frac{1}{\frac{e^{-\tau_e S}}{R_e} + \frac{\bar{w}_s}{\bar{P}_f} S}$
P_{fo}
P_{fi}
BURNER RESPONSE
CONTROL VOLUME RESPONSE
BACK INTO BURNER, NO REFLECTION
0

$^*F_a(i) = K_i e^{-\tau_i S}$

Nomenclature defined on page viii

Figure 4. Model of an air feed system coupled mode of combustion instability in a utility boiler.

5.2.1 Validity of the Analysis

5.2.1.1 Comparison with Previous Analyses

In Appendix B, the final open loop response of the feed system coupled mode of instability analyzed here is compared to more conventional analysis from the liquid rocket engine field. It is shown that the analytical solution derived in this study yields that applicable to a liquid rocket engine when appropriate simplifying assumptions are made. Since the rocket engine chug analyses, such as those presented in previous studies (5), (6) and (7) have been scrutinized, verified and used for many years, the analysis developed here can also be considered generally valid, except where this analysis represents an extension or modification of such analyses to fit the current case.

A block diagram of a typical, simple feed system coupled mode of instability in a liquid rocket engine is shown in Figure 5. Comparison of this diagram with that of Figure 4 broadly indicates the degree of extension or modification made in this study. The seven simplifying assumptions which reduce the block diagram of Figure 4 to that of Figure 5 are:

a. There is no flame in the burner (injector orifice) ($R_{fl} = 0$)

b. The burner (orifice) capacitance is negligible ($C = 0$)

c. The acoustic wave travel time from the region of concentrated combustion to the combustor exit is negligible ($\tau_e = 0$)

d. There is no effect of mixture ratio variations on heat release rates and subsequent pressure variations ($F(r) = 0$)

e. Wave travel times from the burner exit (injector face) to all reflecting surfaces, τ_i, are negligible

f. All acoustic damping and losses due to inefficient reflections are negligible ($K_i = 1$)

g. Perturbations in mass flow from a burner (an orifice) do not immediately appear as mass addition into the combustor volume (implies liquids) but are delayed until the concentrated reaction occurs, at a time τ_c later.

Further comparison of the current analysis with that of a conventional liquid rocket engine analysis can be made by comparing the magnitude-frequency responses of the two analyses for a typical boiler

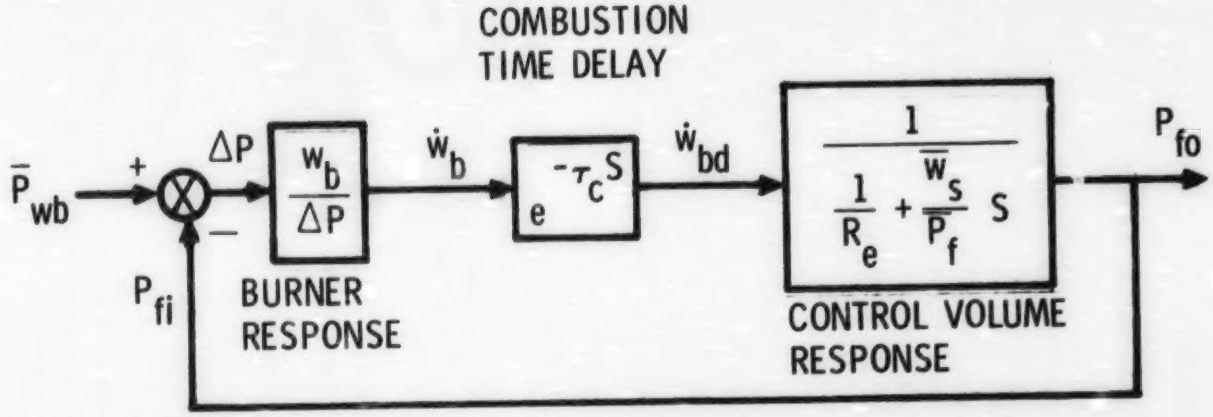

COMBUSTION
TIME DELAY
$\bar{P}_{wb}$
+
−
ΔP
$\frac{w_b}{\Delta P}$
$\dot{w}_b$
$e^{-\tau_c S}$
$\dot{w}_{bd}$
$\frac{1}{\frac{1}{R_e} + \frac{\bar{w}_s}{\bar{P}_f} S}$
P_{fo}
P_{fi}
BURNER
RESPONSE
CONTROL VOLUME
RESPONSE

Figure 5. Typical liquid rocket engine model of a low frequency (chug) feed system coupled mode of combustion instability (simplified from Figure 4).

configuration (defining some of the boiler parameters in terms of their equivalent rocket engine parameters). For all parametric calculations, the geometry of the radiant section of the boiler and the definition of the distances required for wave travel and the reflecting surfaces are defined in the schematic of Figure 6. The remainder of the geometry and nominal full load operating conditions of this typical natural gas-fired utility boiler are given in Table 2.

Table 2. Geometry and Full Load Operating Conditions

L_b = 1.5 m (5 ft)

A_b = 0.5 m^2 (5.24 ft^2)

R_i = 0.083 $sec^2/N\text{-}m^2$ (0.0343 $sec^2/lb_f\text{-}ft^2$)

a = 477 m/sec (1566 ft/sec)

c = 900 m/sec (2950 ft/sec)

$\bar{w}_s$ = 2.39 N (0.537 lb_f)

K_{ri} = 0.9 (for all i)

α = 0

τ_c = 0.04 sec

$\bar{\dot{w}}_b$ = 96.21 N/sec (21.63 lb_f/sec)

$\bar{\dot{w}}_{fb}$ = 10.43 N/sec (2.344 lb_f/sec)

No. of air-only burners = 8

No. of burners, total = 24

Location of air-only burner being investigated:

L_1 = 31.8 m (104.4 ft)

L_2 = 4.15 m (13.6 ft)

L_3 = 3.20 m (10.5 ft)

L_4 = 6.86 m (22.5 ft)

L_5 = 1.58 m (5.2 ft)

L_6 = 7.56 m (24.8 ft)

L_7 = 9.14 m (30.0 ft)

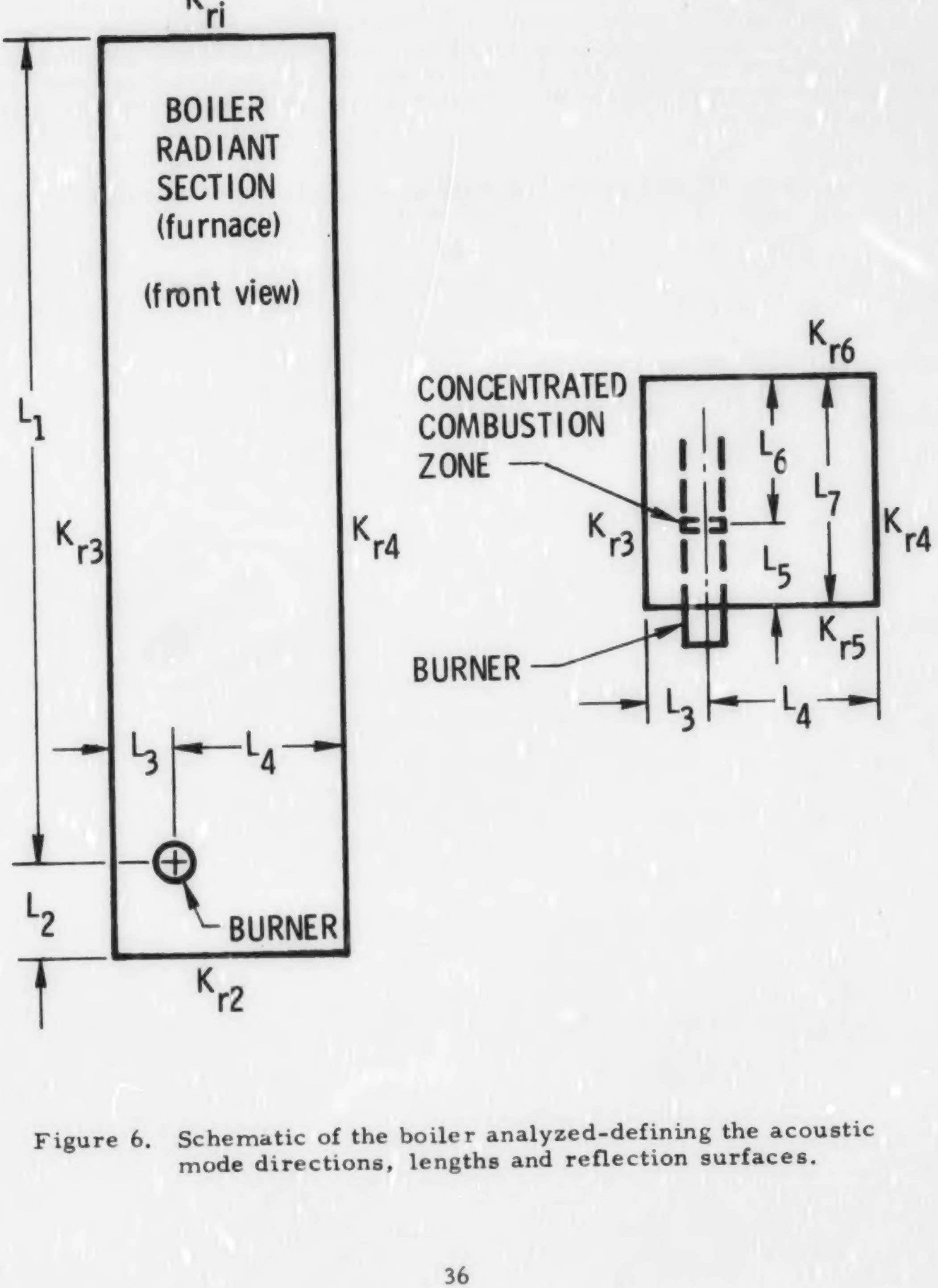

K ri
BOILER
RADIANT
SECTION
(furnace)
(front view)
L 1
K r3
K r4
L 3
L 4
L 2
BURNER
K r2
K r6
CONCENTRATED
COMBUSTION
ZONE
L 6
L 7
K r3
K r4
L 5
K r5
BURNER
L 3
L 4

Figure 6. Schematic of the boiler analyzed-defining the acoustic mode directions, lengths and reflection surfaces.

To compare the more conventional liquid rocket engine feed system coupled mode analysis to that derived in this study, all of the above typical geometry and nominal operating conditions (except $\tau_c = 0.05$ sec) were used to calculate the magnitude of the open loop response over a range of frequencies up to 50 Hz. These results are shown in Figure 7. Noted on the magnitude-frequency curves are the frequencies where the open loop phase shift is 180 degrees. If the magnitude of the response is greater than one at these frequencies, the mode is unstable. The magnitude at other frequencies has little meaning other than to indicate the potential for instability if the phase shift around the open loop were proper (180 degrees). This particular calculation does not indicate the frequency ranges over which the phase shift could ever be 180 degrees.

Clearly the two calculations yield similar results except that the subject calculation appears to superimpose the furnace cavity acoustics on the simpler rocket engine chug analysis. The superposition, however, is actually opposite to what might be expected. Figure 7 shows the frequencies of the various natural resonances of the furnace cavity. The maxima in the response magnitude curve all fall between, rather than at the resonant frequencies. This is the result of the damping effect of the air mass addition at the exit of the burner. As discussed in Section 5.1.2, a decrease in furnace pressure at a burner exit results in an increase in air flow out of the burner. At any of the resonant frequencies of the furnace cavity, or at very low frequency, this increased air flow from the burner will increase the local pressure, effectively with no time delay, which in turn will "fill in the trough" and damp the oscillation. Between the resonant frequencies this effect of mass flow perturbations would be reversed, actually amplifying the open loop response. Figure 7 shows all of these effects, with the minima in the open loop response occurring at zero frequency (out-of-phase) and at all of the resonant frequencies while the maxima occur at off-resonant frequencies.

Figure 7 also shows that a simple, conventional chug analysis would indicate only one instability, a strong one (gain of 3.2) at 6.7 Hz. Based on gain alone, all frequencies below about 24 Hz could be unstable while all above would be stable. The subject analysis indicates a similar relatively strong instability (gain of 1.4) but at a frequency nearly twice as high (11 Hz) as the simpler analysis. The subject analysis also exhibits a gain curve such that instabilities could occur at any frequency below about 22 Hz. Unlike the simpler analysis, however, the subject analysis indicates that the furnace cavity acoustics can modify the gain curve such that instabilities are possible above the 22-24 Hz range. Since this latter prediction results from the modifications introduced in this study, an experimentally observed instability in this boiler in the 40-50 Hz range would tend to further verify those modifications. It will be seen in the following paragraphs that an instability was observed in this frequency range.

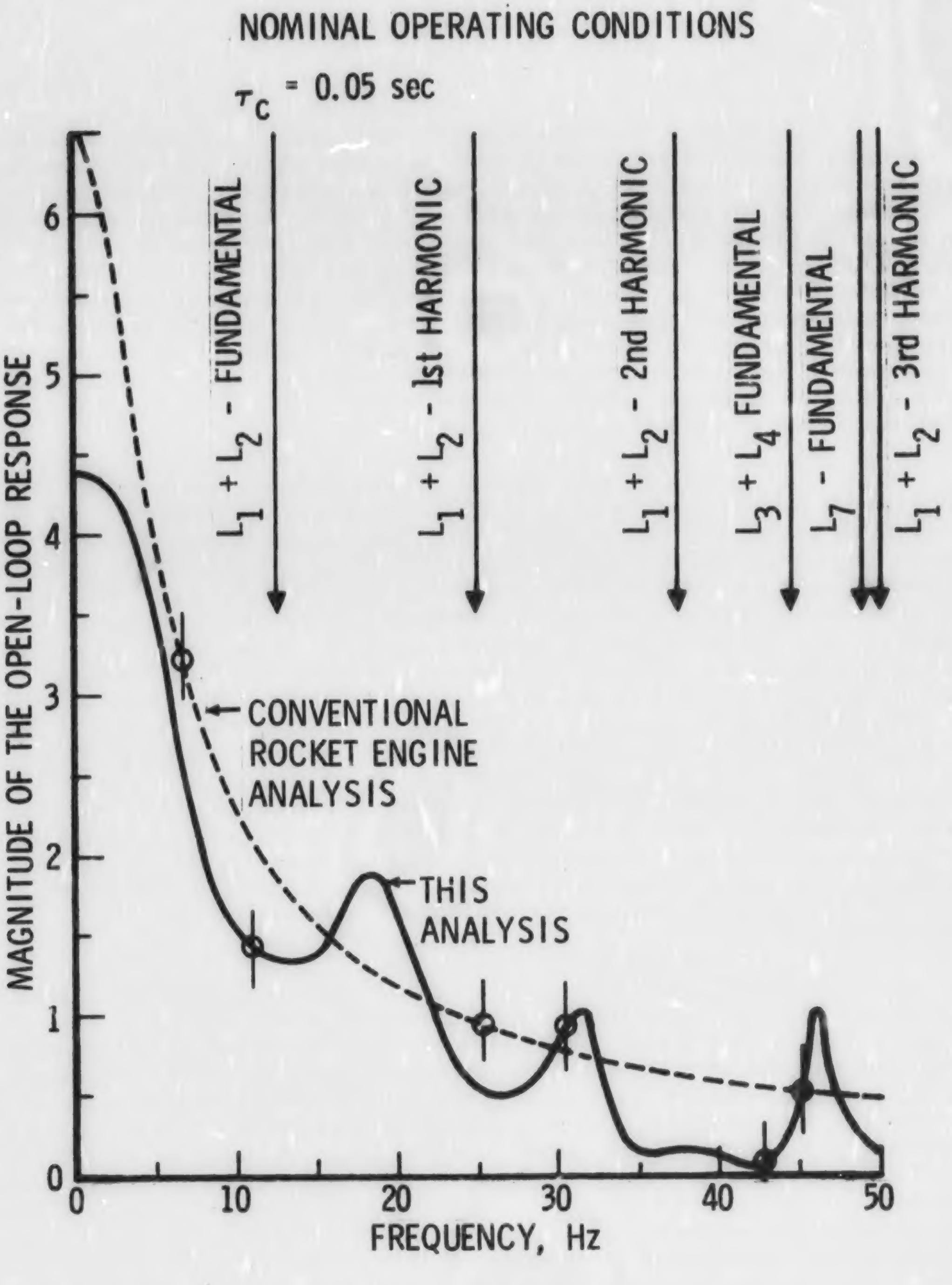


Figure 7. Comparison of results of the current analysis with those of conventional rocket engine analysis.

5.2.1.2 Comparison with Data

The primary data normally used to verify stability analyses are the measured frequencies and relative magnitudes of instabilities, compared to those analytically predicted for those hardware and operating conditions. Obviously, no operator of a full-scale utility boiler wants to operate his boiler unstably in order to develop data to verify an analytical model. The potential danger and financial loss should the instability get out of control is too large. Fortunately for this study, one such case of a violent instability occurred in which not only were the operating conditions reasonably well defined but the acoustic environment (outside of the boiler) was tape recorded for further frequency/amplitude analysis. The major characteristics of this instability were as follows:

a. The most violent instability exhibited maximum amplitude at about 12.5 Hz. Mild vibrations were also observed at about 43 Hz.

b. Maximum instability appeared to occur with 25 to 30 percent of the burners operating air-only. Larger or smaller fractions of burners operated air-only appeared to result in lower amplitude oscillations.

c. All of the instabilities occurred when a special set of gas injection spuds were being used. These spuds were specifically designed to increase the rate of mixing of the gas with the combustion air, very likely resulting in greater combustion within the burners. These gas spuds were subsequently removed from the burners and the instabilities essentially disappeared.

Any other observations are clouded by the fact that the boiler was being operated in a start-up mode at the time of the instabilities with no data taken on total air flow and with little steady operation. A large number of burner configuration, load and other operating condition changes were made simultaneously in an attempt to get the boiler started without damage.

Figure 8 shows plots of the gain curve (magnitude of the open loop response versus frequency) calculated using the subject analysis for all of the nominal hardware and operating conditions in Table 2 except that three values, nominal and ±10 msec, of the combustion time delay are used. These three values are used because the true value appropriate to these combustion conditions is not known, as well as to show that the assumed value of this delay, not at all related to the furnace cavity resonances, has a strong and varied effect on the maxima in the three frequency ranges of interest.

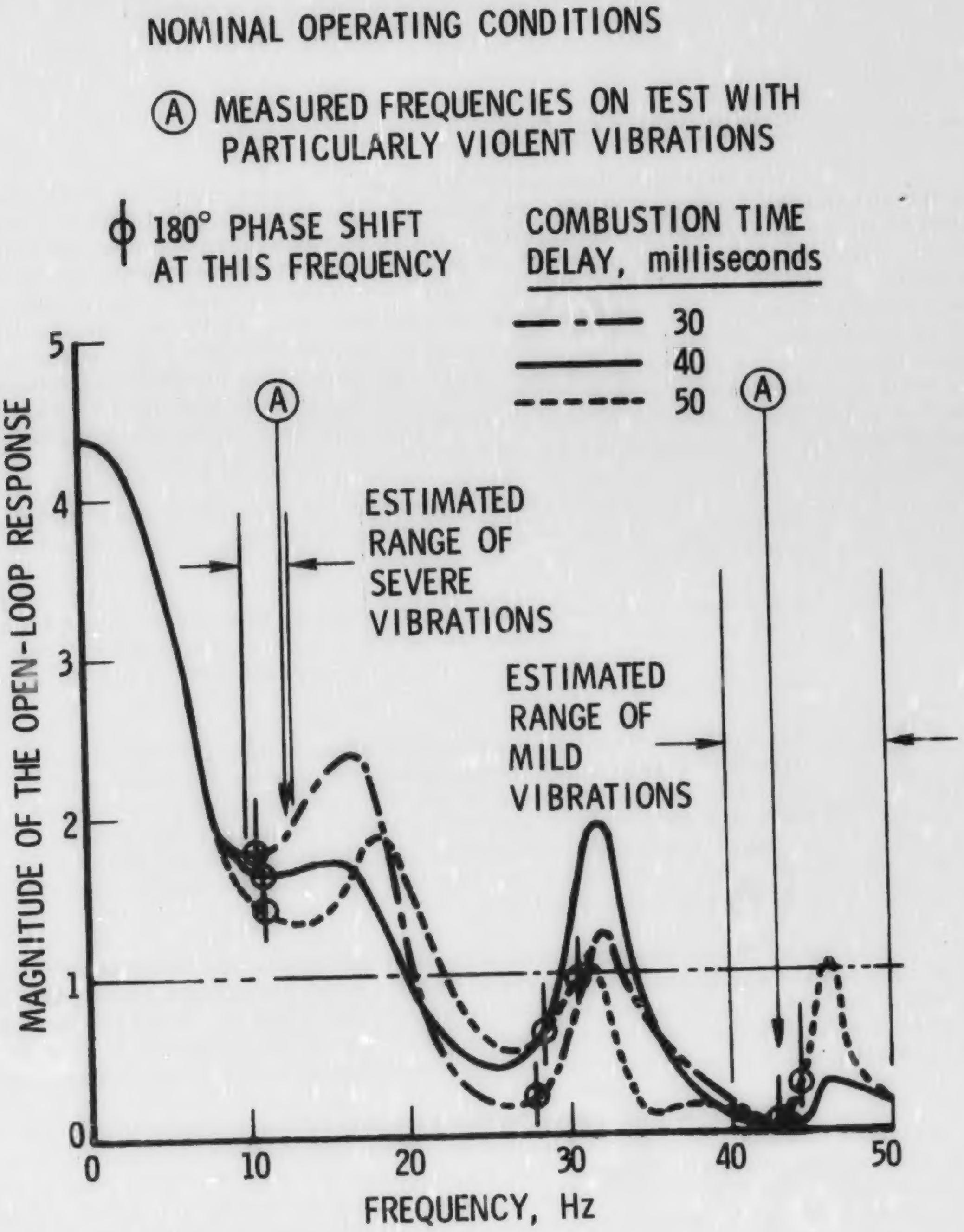


Figure 8. Comparison of analytical predictions with experimental observations.

In the low frequency range, an instability is indeed analytically predicted, in the frequency range around 11 Hz. Frequencies for maximum amplitude were estimated by the engineers in the 10-12.5 Hz range, with the one instability measured in this frequency range at 12.5 Hz. The frequency and stability agreement is considered good, particularly when compared to the 7 Hz instability predicted by the simpler analysis (Figure 7).

In the highest frequency range an instability is not predicted, (the gains are less than one at the frequencies where the phase shifts are 180 degrees) but if the gain were high enough the instabilities would be in the 43-45 Hz range. Instabilities were, in fact, identified by the engineers in the 40-50 Hz range, with the one instability measured in this frequency range at 43 Hz. The frequency agreement is considered excellent. The gain calculation appears to be in error, since it is less than one (stable) at the frequency for 180 degrees phase shift. The calculation does show, however, that a gain greater than one is possible in this frequency range (at 46 Hz) and that the maximum gain in this range is very sensitive to the combustion time delay assumed. It is possible that just slightly different operating conditions could yield a gain greater than one at the appropriate frequency. Figure 7 indicates that the simpler analysis would predict soundly stable operation over this whole frequency range.

Observations from the test where the frequencies were measured indicate that the 12.5 Hz vibrations were very violent while those at 43 Hz were relatively mild. The relative calculated gain in those two frequency ranges are in agreement.

Agreement in the intermediate frequency range (27-36 Hz) is more questionable. No experimental estimates or measurements indicated any instabilities in this frequency range. The calculations, although they do not actually indicate an instability, do indicate a potential for instabilities intermediate in both magnitude and frequency between the ranges discussed above. Figure 7 shows that in this intermediate frequency range relatively high gain results from the first and second harmonics of the vertical ($L_1 + L_2$) mode while the gain maxima in both the lower and higher frequency ranges result from fundamental resonances.

The calculations shown in Figure 8 represent the response of a burner very low in the furnace cavity (see L_1 and L_2 in Table 2), where the pressure oscillations in all of the vertical resonant modes of the furnace cavity, and the pressure coupling with burner flow, are maximum. In the actual case the burners are spread vertically from about 10 to 30 percent of the vertical height of the furnace cavity. Figure 9 shows that the loop gain is maximum in both the low and intermediate frequency ranges when the burner is located at the lowest level. In the lower frequency range (15-22.5 Hz

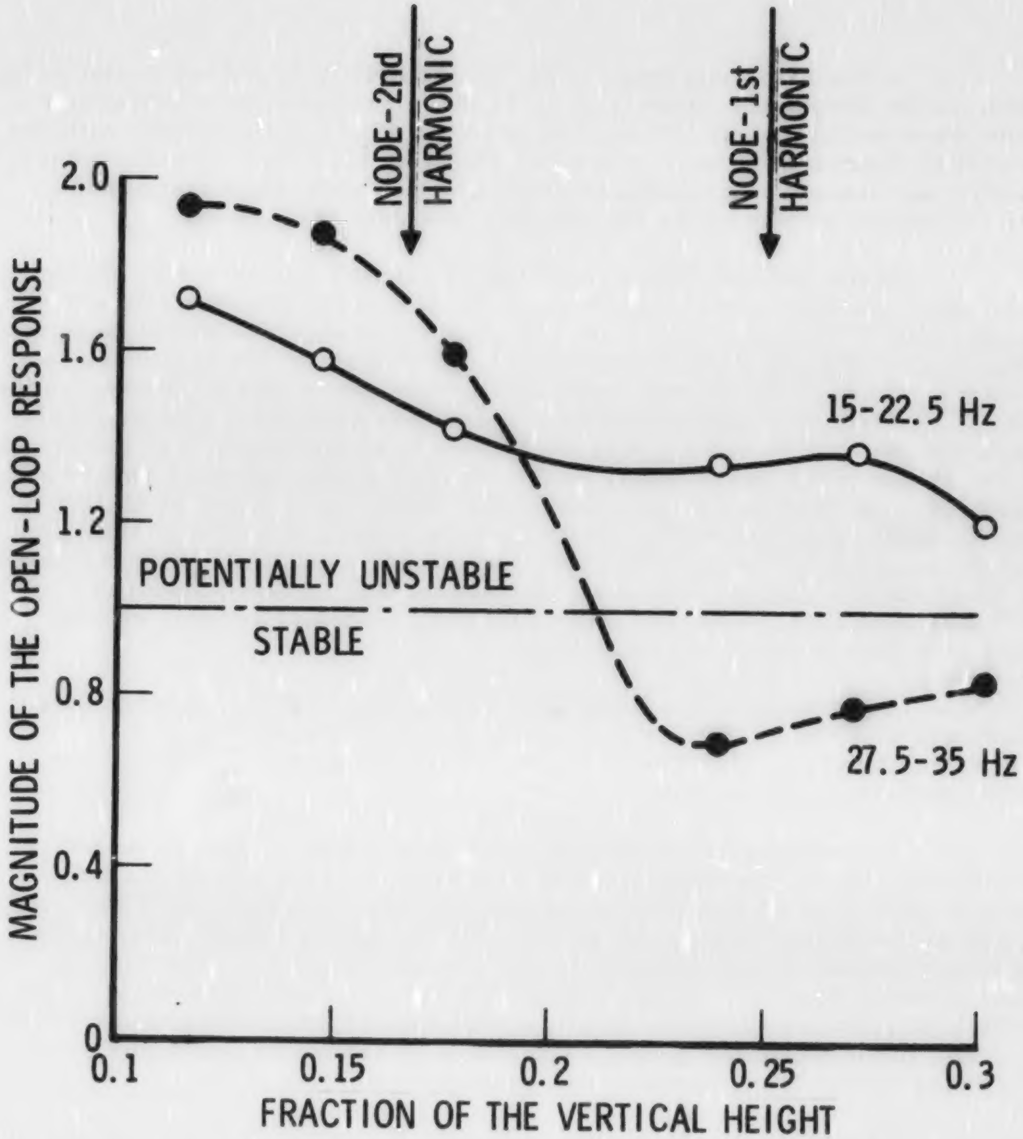


Figure 9. Variation of the maximum gain within the low and intermediate frequency ranges for burners located at various vertical positions

between the fundamental and the first harmonic of the vertical resonance) the response decreases with increasing vertical height but is always relatively high and always potentially unstable. In the intermediate frequency range (27.5-35 Hz), between the first and second harmonics, the loop gain decreases rapidly in the higher levels and burner locations above about 0.2 of the vertical height are stable in this frequency range. As a result, the appropriate total response of all of the active burners in a real boiler may always be less than one (and stable) in this intermediate frequency range. This interpretation would be in agreement with the limited experimental observations with this boiler.

The variation in loop gain for burners at various vertical positions can be explained simply from the pressure coupling. Since it is the furnace pressure oscillations at the location of a burner exit that cause the variations in air flow through the burner, pressure coupling is greatest at the pressure anti-nodes of the resonant mode and is zero at the nodes. Table 3 lists the locations of the anti-nodes and nodes in the first three of the vertical resonant modes in the furnace cavity.

Table 3. Anti-Nodes and Nodes in the Vertical Resonant Modes of the Furnace Cavity

Fraction of the Vertical Height	Pressure Anti-Node (A) or Node (N)		
	Fundamental	1st Harmonic	2nd Harmonic
0	A	A	A
1/6	-	-	N
1/4	-	N	-
1/3	-	-	A
1/2	N	A	N
2/3	-	-	A
3/4	-	N	-
5/6	-	-	N
1	A	A	A

The lowest pressure nodes in the first and second harmonics of the vertical mode are also shown in Figure 9. The node between these two harmonics, then, should fall at about 21 percent of the vertical height, approximately at the location, in Figure 9, of the minimum in the gain in that frequency range. The average node between the fundamental and the first harmonic, however, is off scale in Figure 9, at about 37.5 percent of the vertical height. Nevertheless, the figure shows the gain in the lower frequency range decreasing toward that node.

As a result, it seems reasonable to interpret the above calculations to indicate that the net total response of an array of burners spread over a vertical distance large compared to the wavelength of the 2nd harmonic of the vertical resonance would tend to be stable in the frequency range between the first and second harmonics. It is also reasonable to imply that this vertical spreading of the burners would have little effect on stability in the frequency range between the fundamental and first harmonic resonances (15-22.5 Hz) and no effect on stability in the frequency range of the transverse (horizontal) resonances (45 - 49 Hz). Experimental data confirms all of these observations.

As a further check that the gain maxima discussed here and shown in Figure 9 are indeed related to the vertical acoustic resonances, the acoustic reflection coefficients at the top and bottom of the furnace, affecting only the vertical mode, were analytically set to zero. These results are shown in Figure 10. Clearly the absence of acoustic reflections severely reduces the low and intermediate maxima.

It may well be that these observations on the relative stability of this boiler in these three frequency ranges represent the most fundamental verification of the subject analysis. The analysis not only reasonably accurately predicts the frequency ranges and relative stability of the low and high frequency instabilities but also correctly predicts the relative stability in the intermediate frequency range. The only alternative explanation known to the writer for the absence of instability in the intermediate frequency range might be that related to the Helmholtz acoustic absorber represented by the ash pit of this boiler (1). Total (analytical) elimination of reflections from the bottom of the furnace, however, still leaves a fairly high and potentially unstable gain in the intermediate frequency range.

The second source of experimental verification of the subject analysis is the previously discussed observation that the maximum instability appeared to occur with 25 to 30 percent of the burners operating air-only. The fraction of the burners which are operated on air-only has a strong effect on NO_x emissions. Therefore, that parameter will be discussed in the next section (5.2.2), where the effects of major combustion modifications on stability are evaluated. The reason that the boiler appeared to operate

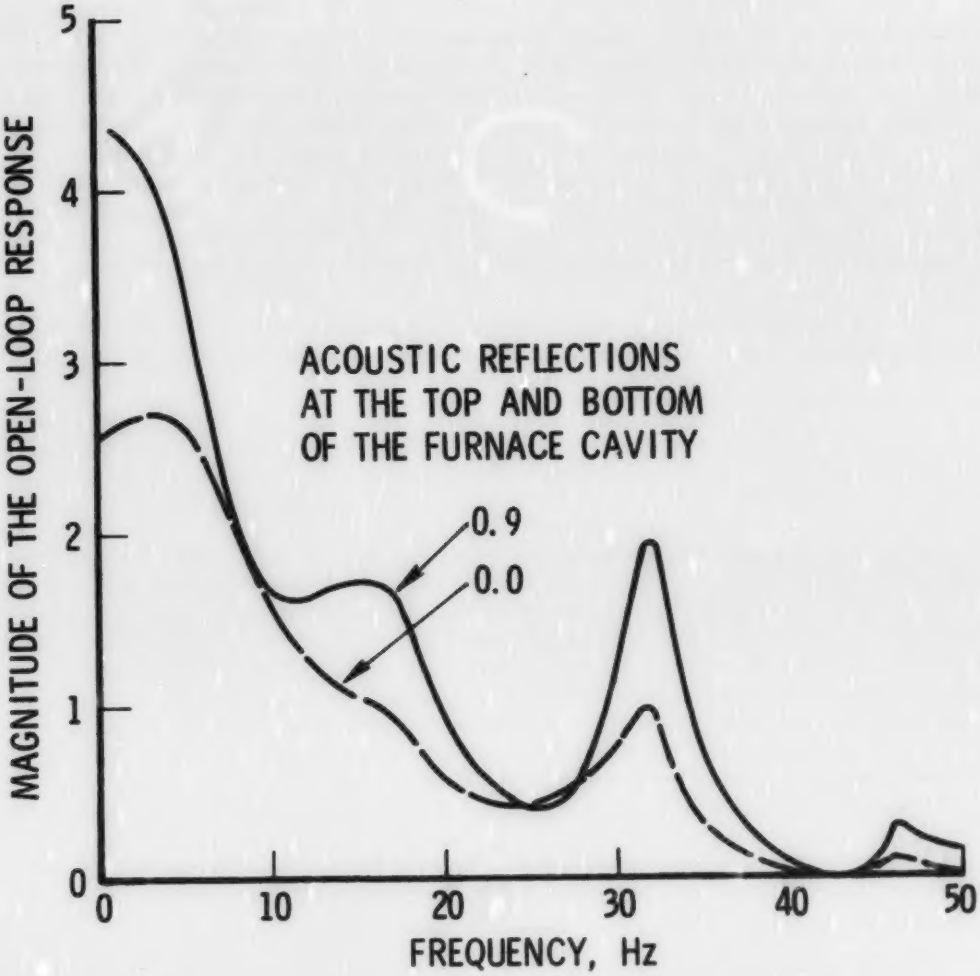


Figure 10. Effects of acoustic wave reflection efficiency.

somewhat more stably when more than 25 to 30 percent of the burners were operated air-only is thought to result from the more stable position of the flame, deep within the burner.

This same variable, the fraction of combustion completed within the burner, is also thought to be at the root of the violent instability in this boiler in the first place. All of the natural gas burner spuds were changed just prior to the observed instabilities to incorporate a new design specifically intended to promote rapid mixing of the fuel with the air within the burner. It seems reasonable that this more rapid mixing would cause more of the combustion to be completed within the burners. No other change or effect was apparent. After two attempts to start the boiler and to achieve rated load, during which the violent instabilities occurred, the original gas spuds were returned to the burners and stable operation was restored.

In the subject, as in the previous (1) analysis, the fraction of combustion occurring within the burner was described by a function of the type:

$$C_n = K_3 \dot{w}_b^{-n} \tag{1}$$

where, with the standard gas spuds, K_3 and n were estimated at 0.777 and 0.5, respectively. Equation (1), with these values for the constants, was used for nearly all of the parametric stability calculations. It is not known exactly how the form of Eq. (1) might change with more rapid fuel-air mixing within the burner. It seems reasonable, however, that a flame which is long compared to the burner length (from the gas spuds to the exit) would not move substantially in and out of the burner with burner flow velocity changes. Conversely, a flame which is short compared to this burner length might easily move completely inside the burner with normal burner flow velocity variations. Thus Eq. (1) for the slow-mixing flame might involve a nearly zero exponent (-n) while a rapid mixing flame might involve much more negative slopes (larger values of n). For very rapid mixing the flame might be completely contained within the burner at all flow velocities and the exponent (n) might again be zero.

A series of parametric calculations were made to investigate the effect of the negative slope of Eq. (1) on stability. For these calculations the total burner resistance at a given burner air flow rate was held constant while the exponent n was varied from 0 to 2. The value of the constant, K_3, then, also had to be varied.

Figure 11 shows plots of the low frequency open loop response for values of the exponent ranging zero to two. At very low frequency (near zero) the effect of the exponent variation is to increase the response by more

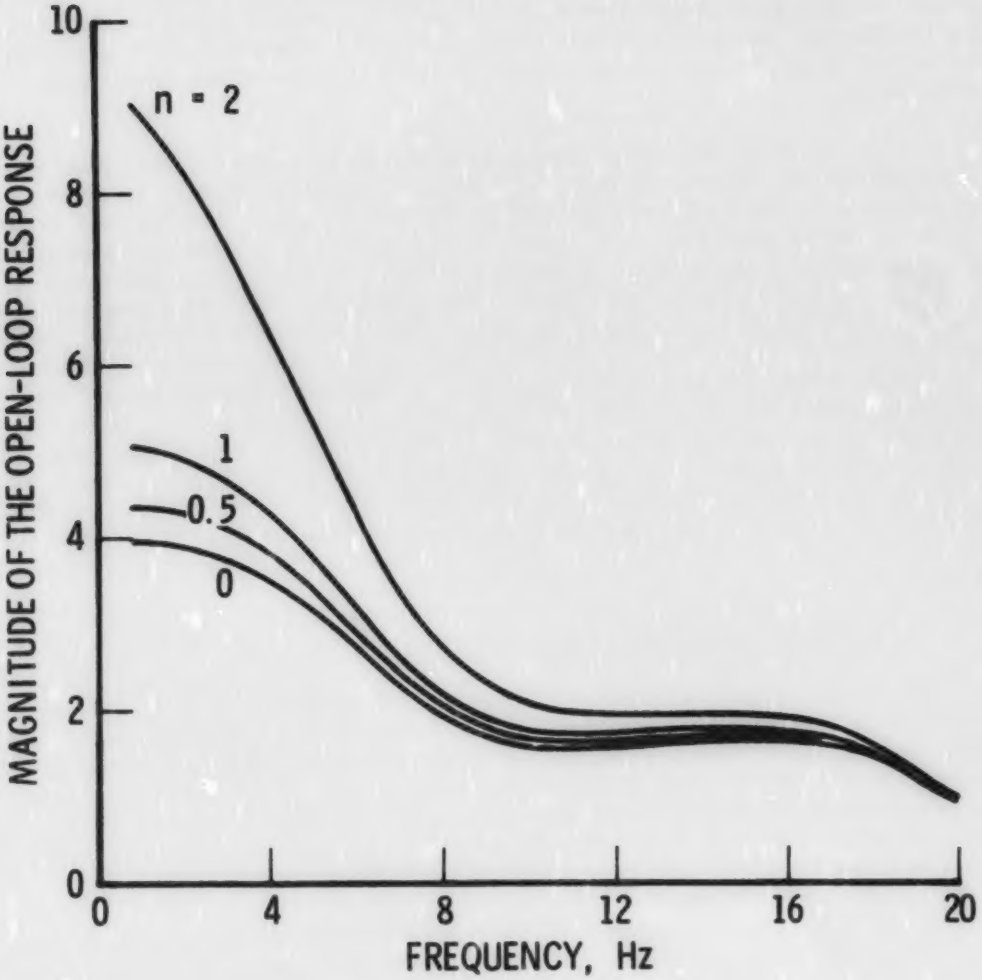


Figure 11. Effects of the fraction of combustion completed within an active burner on combustion stability.

than a factor of two as the value of the exponent was increased from zero to two. At the measured unstable frequency (12.5 Hz) the increase in response is only about 25 percent. With an exponent of zero, which implies that air flow velocity changes would have no effect on burner resistance or stability, the figure shows that the open loop is still potentially quite unstable (gain greater than one).

In general the subject analysis tends to support the observations regarding the effect of combustion within a burner. First, in cases where combustion is partially completed within the burner, the burner boiler-loop tends to be more unstable. Second, in cases where combustion is almost totally completed outside or downstream of a burner (oil- and coal-fired boilers) or inside of a burner (very rapid mixing with very low air flow velocity) the boiler tends to be more stable. The relatively high gain, relatively unstable system predicted even with the zero exponent is somewhat surprising since instabilities are often observed with the fast-mixing and burning gaseous fuels but almost never with the oil or coal fuels. This writer, however, is aware of at least one oil-fired boiler which not only exhibits repeatable instabilities but the instabilities appear to exist only when the flame is visually observed to be partially within the burner (rather than distinctly separated from, and downstream of, the burner). There appears to be more to learn about the effect of this parameter on stability of utility boilers.

5.2.2 Further Parametric Calculations

Some of the parametric calculations made in this study have been discussed in the previous section. The purpose there was to compare results with experimental observations to confirm the validity of the analysis. In this section it will be assumed that the comparisons of the previous section have reasonably validated the analysis and the results of some of the parametric calculations will be reviewed for the purpose of evaluating and explaining the effects of certain independent variables on stability.

The primary purpose of the overall EPA grant, of which this analysis of combustion instability in utility boilers is a part, is to evaluate practical techniques for the control of NO_x emissions by combustion modification. Stable combustion is a necessary requirement over the full range of modified combustion conditions. In previous studies (1) and (2), it was determined that the single most effective combustion modification technique for NO_x control with natural gas, oil or coal fuels is to concentrate the fuel flow into a fraction of the burners, letting the remainder of the burners (and, perhaps, special NO_x ports) operate with air-only. The active burners, then, would operate very fuel-rich. These previous studies have also indicated that the fuel-rich active burners should be located low in the burner array for maximum NO_x reduction. There are also indications from other studies that fuel-air mixing within the burners can have a significant effect on NO_x. For these reasons, the effects of three major combustion modifications on combustion stability are discussed in this section: (a) the fraction of burners operated air-only; (b) the vertical location of air-only burners, and (c) the fraction of combustion completed within a burner. For convenience, NO_x ports will be considered an equivalent air-only burner.

5.2.2.1 The Fraction of Air-Only Burners

The main effect of shutting off the fuel to some of the burners and diverting it to others is to provide an initial fuel-rich region in the boiler in which the initial hydrocarbon-air reactions can take place. This minimizes both the conversion of fuel-bound nitrogen to NO_x as well as the formation of NO_x by thermal mechanisms, at least in this initial combustion stage. Indications from the studies summarized in Ref. 3 are that it is desirable to operate the active burners as fuel-rich as possible, consistent with the flammable limit and with stable combustion.

The main effect of the burner air-fuel ratio on combustion stability is in the amplification of heat release and mole change perturbations resulting from air-fuel ratio perturbations at mean burner air-fuel ratios well away from stoichiometric. This amplification is discussed in Appendix B and is represented by the function F(r), defined by Eq. (B-55). Equation (B-73) shows that F(r) has no significance at very low frequencies (approaching zero) but can represent a significant gain term, or destabilizing influence, at the frequencies of interest here. Figure 12 shows a plot of the approximation analytical expressions used in this study to represent F(r). The data for these approximations were derived from equilibrium combustion calculations performed at The Aerospace Corp.

The figure shows that values of F(r) apparently become strongly positive at air-fuel ratios lower than about 90 percent of stoichiometric (fuel-rich). It seems likely that the value of F(r) could even become infinite as the flammable limit is approached, although the equilibrium combustion calculations could not predict this. Depending on mixing within the burner and the recirculation necessary to anchor (provide continuous ignition for) the flame, the flame could repeatedly blow off the burner (lift off) and flash back or re-attach. In such a case, the heat release at the burner exit would alternate between zero and some significant, finite level. Equations (B-76) through (B-87) in Appendix B show that for very large values of the function F(r), the magnitude of the open loop response is directly proportional to F(r) and the open loop response would in turn be very large.

Thus, the onset of instability and the incidence of flame lift-off could both be rather simply related to a burner air-fuel ratio, near to the flammable limit. For natural gas, the fuel-rich flammable limit is an air-fuel ratio about 62 percent of stoichiometric. With a boiler operating at

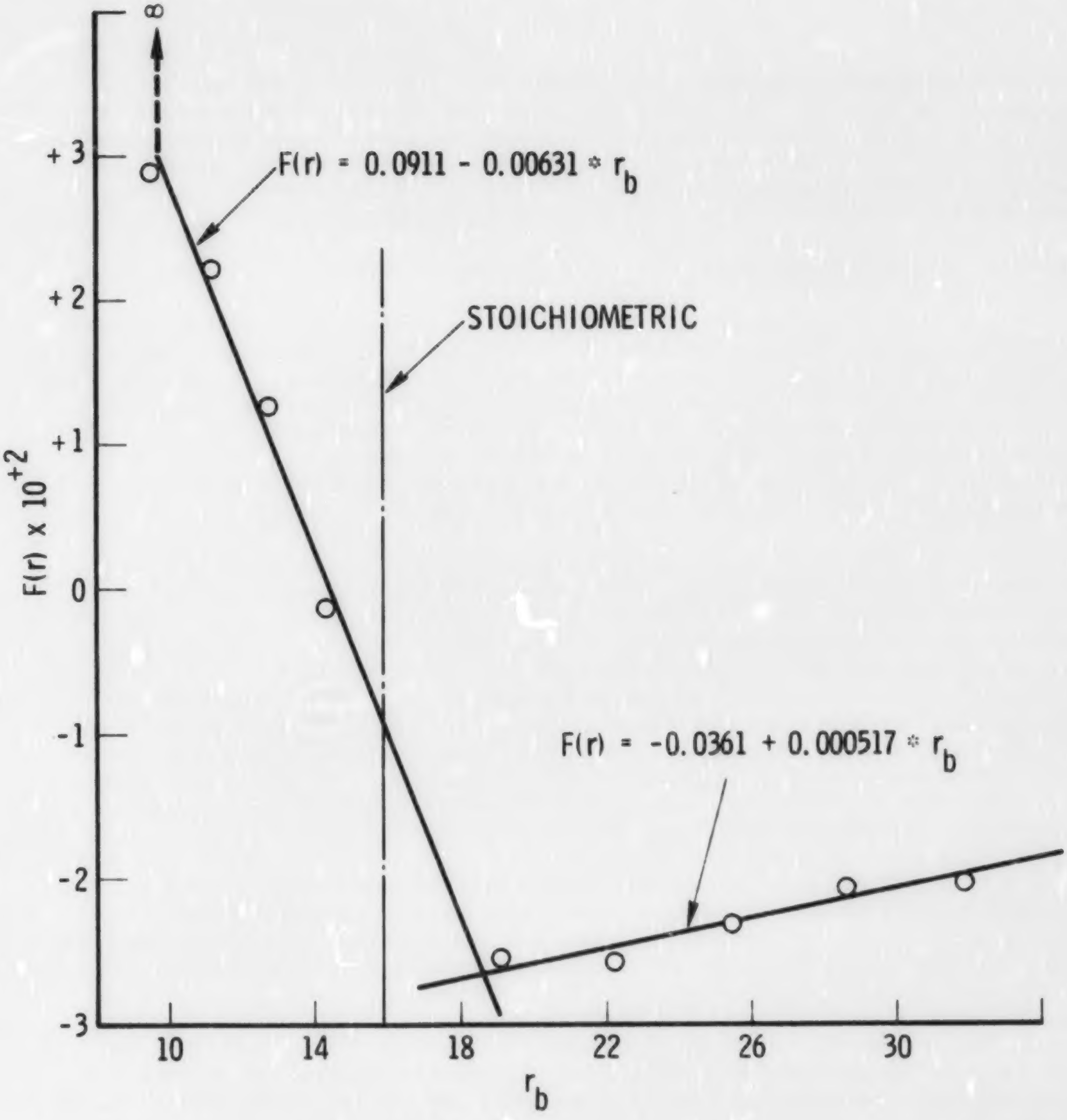


Figure 12. Analytical expressions used to approximate the function F(r).

about 15 percent excess air, and taking into account the increased resistance to air flow as a result of partial combustion within the active burners, the rich flammable limit could be approached in the active burners with about one-third of the burners operated air-only. Vibration data from two boilers firing natural gas fuel appear to indicate an onset of undesirably high vibrations with about 25 to 30 percent of the burners air-only. Some subjective observations of the flames at the burner exit operating under these conditions, however, tend to indicate that combustion is in fact occurring at air-fuel ratios very near to the rich flammable limit (1). Figure 13 shows an estimate of the actual burner air-fuel ratio, as a function of the fraction of the total burners that are active, for the boiler used as the nominal in this study.

Figure 12 also shows that fairly large negative values of F(r) are calculated at air-fuel ratios typical of the overall boiler (or of the burners when all burners are active and no NO_x port air flow exists). Operation at about three percent oxygen in the flue gases represents about 15 percent excess air. At low frequencies the negative gain would tend to have a stabilizing effect but at higher frequencies the effect would depend on other phenomena such as the acoustic time delays and could be destabilizing. At very high (lean) air-fuel ratios, regardless of whether the flameholding is adequate, the values of F(r) begin to approach zero and additional gain from this source tends to become negligible. The large dilution of any heat release and mole change variations by the excess air very likely damps any subsequent destabilizing effects.

If one neglects the extreme cases of F(r) that might be possible at air-fuel ratios near the rich flammable limit, the effects of the fraction of the burners operated air-only do not appear large. Figure 14 shows that the open loop gain at the unstable frequency (10-11 Hz) increases as the fraction of the burners operated air-only increases but over the range from zero to about 38 percent of the burners air-only this unstable response increases by only about 15 percent. Similarly, Figure 15 shows that although increasing the fraction of air-only burners from zero to half increases the maximum gain in the 30-35 Hz range by more than 80 percent, it actually decreases the maximum gain in the 15-20 and 45-50 Hz ranges.

Thus, it appears that one of the major combustion modification techniques for the control of NO_x can be accomplished without significantly increasing the potential for combustion instability, as long as the burner air-fuel ratio is maintained above the rich flammable limit. It is probable that the burner air-fuel ratio, on the average, could be safely reduced even below the rich flammable limit if care were exercised to maintain a solidly anchored flame, avoiding any flame liftoff.

5.2.2.2 Vertical Location of Air-Only Burners

Some of the parametric calculations regarding the effect of this parameter have been shown and discussed in Section 5.2.1. Figure 9

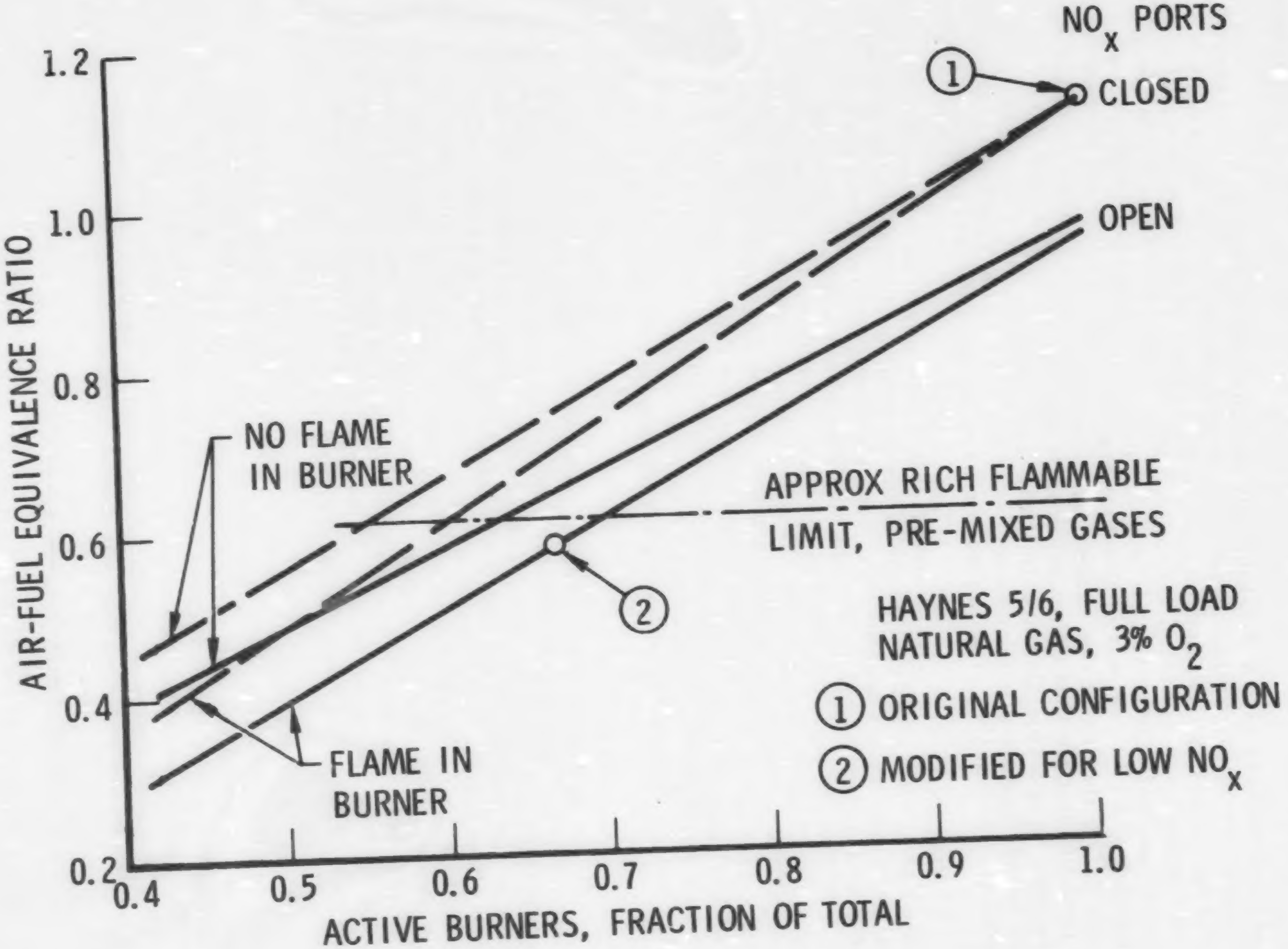


Figure 13. Burner air/fuel equivalence ratio with/without flames in burners.

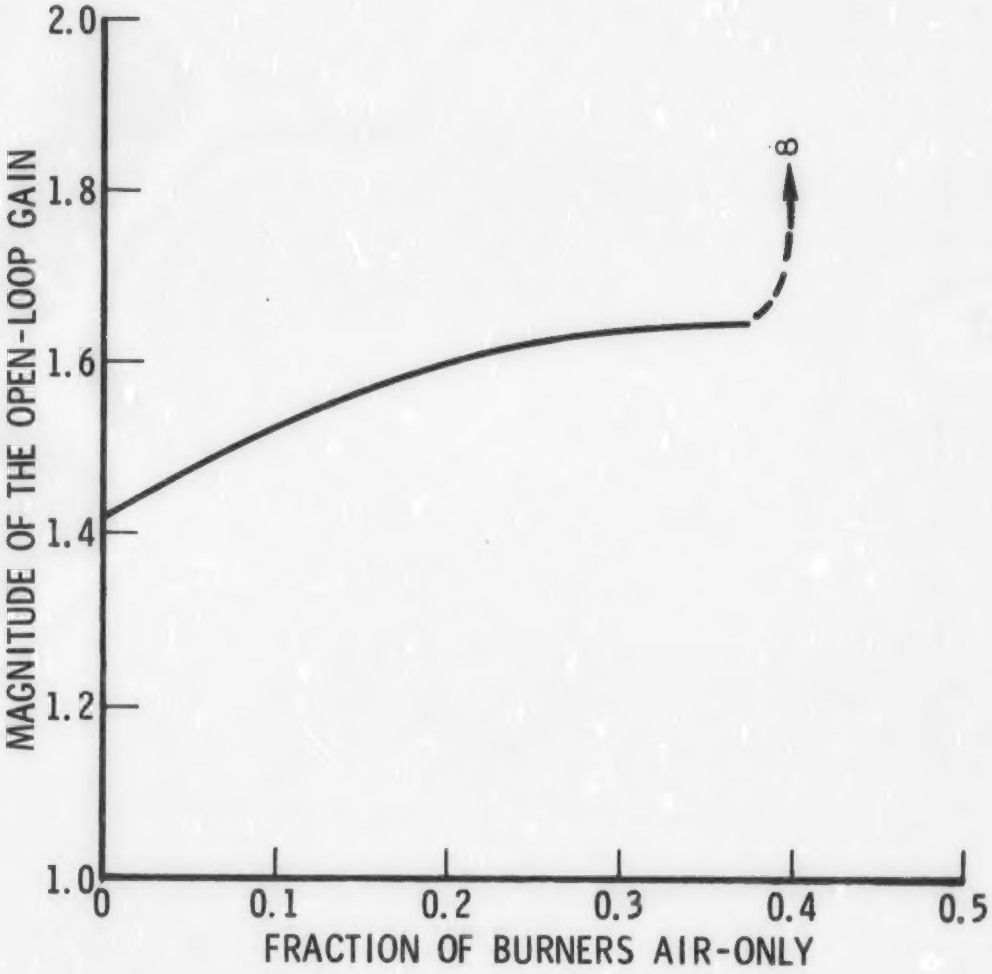


Figure 14. Effects of the degree of burners-out-of-service on instability at the unstable frequency.

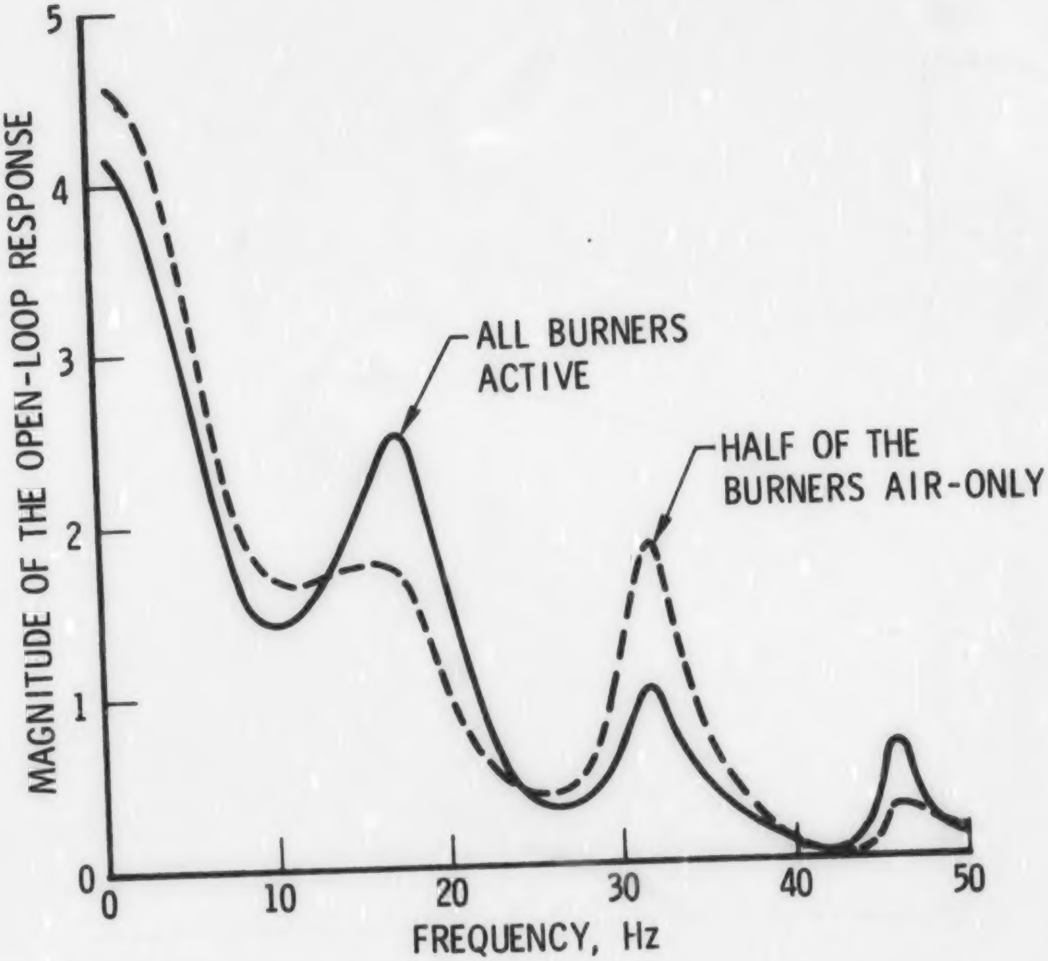


Figure 15. Effects of the degree of burners-out-of-service on instability at various frequencies.

for example shows that the maximum gain of the open loop occurs when the active burner is located as low as possible in the burner array. As discussed previously [(1) and (2)] this is exactly where the active burners should be to minimize NO_x emissions, at least in the case of oil and coal fuels. Dykema (1) discusses the possibility of minimizing NO_x emissions when burning gaseous fuels (containing no fuel-bound nitrogen) by locating all of the air-only burners as low as possible in the burner array or by providing a configuration with all air-rich (fuel-lean) active burners in conjunction with fuel-rich NO_x ports. Although both of these latter configurations have only been suggested, and have not been adequately demonstrated, both represent more stable configurations than those with fuel-rich active burners in the bottom of the burner array. In cases where the latter configuration is necessary, care will have to be taken to maintain combustion stability.

5.2.2.3 Flame Within the Burner

Operation of a boiler with all or part of the combustion taking place within the burners is not a consciously recognized NO_x reduction technique. From study of both the steady state and the vibration data of Ref. 1 it appears that combustion within a burner has a significant effect on both NO_x emissions and on combustion stability.

At least some initial combustion in the burner or near the burner exit is necessary to provide an anchor, or a continuous ignition source, when the fuel is gaseous. It may also be desirable in some oil-fired configurations, particularly when atomization is fine and vaporization and burning is rapid.

The discussion in Section 5.2.2.1 and in the previous study (1), as well as the calculations plotted in Figure 13, show why it may sometimes be thought desirable to maintain a significant flame within the burners. The primary effect of combustion within a burner is to increase the resistance to air flow through the burner which, in turn, reduces that air flow rate and decreases the burner air-fuel ratio. In general, a lower burner air-fuel ratio reduces NO_x emissions. As a result, low NO_x emissions are observed with the flame in the burner; high NO_x emissions are observed with the flame outside of the burner and the general conclusion is (sometimes) drawn that a flame is necessary within the burner to achieve low NO_x emissions.

On the other hand, the incidence of high combustion vibration is much higher when a partial flame is anchored within a burner. Nearly all cases of excessive vibrations of the type described herein, at least within the experience of the writer, have occurred in natural gas-fired boilers, where the flame almost must be anchored in the burner. In the one (undocumented) case of such vibrations in an oil-fired unit of which the writer is aware, the vibrations reportedly were present when the flame was said to be in the burner exit but absent when the flame was somewhat downstream from the exit. From the standpoint of combustion vibrations, then, it might be concluded that it is necessary to avoid the presence of a flame within a burner to avoid excessive combustion vibrations.

The calculations shown in Figure 13, however, show that a flame need not be present within a burner to achieve a given active burner air-fuel ratio and the related NO_x reduction. The effect of flames within the active burners is to increase the resistance to air flow through the active burners relative to the air-only burners and to divert combustion air flow away from the active burners and through the air-only burners. This also increases the overall resistance of the total burner array, requiring more fan energy to drive the combustion air into the furnace. Often, attempts to reduce NO_x emissions by concentrating the fuel flow in a fraction of the burners result in "running out of fan" - rated load cannot now be achieved because the combustion air fans cannot drive air across the burner array at that rate.

With no flames within the active burners, the air flow resistances of active and air-only burners and the overall resistance of the total burner array are essentially independent of the fuel flow rates in the individual burners. The fuel flow must now be concentrated in a smaller fraction of the burners to achieve the active burner air-fuel ratio, but once this is done the NO_x reduction should be the same (disregarding the slightly different mixing and burning pattern in the furnace with the fuel concentrated into a smaller number of active burners).

Most of the above steady-state effects of partial combustion within a burner were first derived and discussed in the studies reported by Dykema (1). The subject study has added to this some understanding of the dynamic effects. Generally, the presence of a flame within an active burner represents a destabilizing influence on the boiler combustion. This is particularly true if the active burner air-fuel ratio is reduced to near the rich flammable limit of the fuel. The studies reported here, however, show that the presence of the flame within the burner does not guarantee unstable or vibratory combustion.

A particularly important result of this study is the observation that the absence of any influence of the flame on air flow through the active burners does not in itself assure stable combustion. Note that the open loop response shown in Figure 11 for this case (n = 0) still shows significant potential for instability (response greater than one). Care must be taken to design for and avoid combustion instability in all cases, even if the flame is fully outside of the burner and/or solidly anchored. This also implies that similar instabilities can occur in oil- and coal-fired boilers as well.

Generally the presence of a flame within the active burners has two negative effects: (a) it has a destabilizing effect, particularly as the active burner air-fuel ratio is reduced near to the level of the fuel-rich flammable limit for the fuel; and (b) it unnecessarily increases the overall resistance to air flow through the total burner array. Since there appear to be no direct compensating positive effects, (steady-state air flow resistance can be more reliably controlled with the inlet resistance) it is probably best to design for minimum or no flame within the active burners. As in all combustion systems (8), there is no dynamic substitute for high, constant (independent of fluid flow) resistance across the burners to maintain stable combustion, but this is probably better achieved mechanically than as a result of partial combustion within the active burners.

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APPENDIX A

EFFICIENCY LOSSES

Five areas of efficiency losses are addressed here, representing energy losses resulting from:

a. Incomplete combustion

b. Uncondensed water vapor in the flue gases

c. The high temperature of the flue gases entering the stack

d. Steam turbine expansion processes

e. Electrical generating and control equipment

The reference input energy rate is taken as the high heat of combustion. By definition, the high heat of combustion is released and available to do work in a boiler only if all the carbon and hydrogen in the fuel are oxidized to carbon dioxide and water and the resulting combustion products are cooled to their initial ambient temperature. The first three of the above losses represents departures (heat losses) from this ideal. When these heat losses are subtracted from the high heat input rate, the remainder is essentially the rate at which heat enters the boiler steam cycle.

The electrical equipment is also not 100 percent efficient. After subtracting these electrical losses from the measured electrical load, the remainder is essentially the energy output rate of the steam turbine. The ratio of this output rate to the rate at which heat enters the boiler steam cycle represents a rough measure of the steam, or Rankine, cycle efficiency of the boiler. The Rankine cycle of a modern utility boiler is usually complicated by various reheat and regenerative partial cycles and other means designed to maximize performance. Since the prime purpose of this study was and is NO_x control, not enough data were gathered to allow detailed direct evaluation of losses, and changes in losses with combustion modifications, throughout the steam cycle.

The largest single steam cycle efficiency loss is that inherent in the thermodynamic process (ideal cycle efficiency). We have no interest in this loss in this study except to note that it will be different for boilers designed for different steam pressures and temperatures. The next largest loss, however, is usually a result of nonadiabatic expansion through the steam turbine. In most cases, the steam turbine is designed for maximum efficiency in the expansion process under the most common operating condition and is less efficient under other conditions. For a utility boiler, it is reasonable to expect that maximum expansion efficiency may occur at about 80 percent of rated load. Efficiency losses at other loads largely result from the need to control the power output of the fixed geometry, constant speed turbine by controlling the thermodynamics of the steam. It is also reasonable, then, to assume that a large part of the variation in efficiency of the turbine with load would result from the steam power control technique (for example, throttling), essentially independent of the profile of the heat flux into the steam. These losses would result from the need to vary the boiler load (electrical output) independent of any combustion modifications introduced for the purpose of controlling NO_x.

Thus, by calculating efficiency losses in four of the five areas mentioned above, the major known losses can be accounted for, leaving a residual efficiency which can contain all the remaining variations in efficiency which cannot easily be analyzed. The efficiency loss calculations are developed in Sections A.2 through A.5 of this appendix.

A.1 STOICHIOMETRY

Before many of the efficiency losses for which we want to account can be calculated, various quantities must be derived from stoichiometry. These quantities are derived in this section and cast in a form for calculation by computer.

Writing a generalized hydrocarbon fuel molecule as $C_aH_bO_cN_dS_e$ and considering only the major combustion products or those of special interest to efficiency or air pollution, stoichiometry can be written

$$(C_aH_bO_cN_dS_e) + X(O_{0.42}N_{1.57}) \rightarrow A(C) + B(CO) + C(CO_2)$$
$$+ D(H_2O) + E(O_2) + F(N_2) + G(NO) + H(SO_2) \qquad (A-1)$$

It would probably be desirable to include free hydrogen in the products, but no data are available on hydrogen concentrations in the flue gases.

Samples of the combustion products are usually analyzed after all the water vapor has been condensed and the remaining gases thoroughly dried (dry basis), and concentrations are normally cited as volume or mole fractions of the total sample. The product specie concentrations then can be defined by (e.g., for CO_2)

$$[CO_2] = \frac{C}{SMTD} \qquad (A\text{-}2)$$

where the square brackets [ ] denote a concentration, SMTD is defined as the sum of the moles of combustion products, excluding the water (dry) per mole of fuel, and C is the coefficient of CO_2 in Eq. (A-1). Equation (A-2) defines that the concentration represents a volume or mole fraction, dry.

There are three parameters of interest to efficiency and air pollution studies which can be derived from flue gas analyses using Eqs. (A-1) and (A-2):

a. SMTD

b. Overall air-fuel ratio

c. Heat of combustion

The theoretical stoichiometric relation of CO_2 to O_2 is also useful. From material balances in Eq. (A-1), expressions for these parameters are

$$SMTD = \frac{NUM}{DENOM} \qquad (A\text{-}3)$$

$$NUM = 0.9406b - 1.8811c + 0.5d + 4.7623e \qquad (A\text{-}4)$$

$$DEMON = 1 - 2.8811[CO] - 4.7623[CO_2] - 4.7623[O_2] - 2.388[NO] \qquad (A\text{-}5)$$

$$X = 2.3821\left\{\left(\frac{1}{2}b - c + 2e\right) + \left([CO] + 2[CO_2] + 2[O_2] + [NO]\right)SMTD\right\} \qquad (A\text{-}6)$$

$$\begin{aligned}\Delta H_{ch} = &-\left(\Delta H^{o}_{ff} + 34.20b + 69.3e\right) - \left(26.42[CO] + 94.38[CO_2]\right.\\ &\left.- 21.5[NO]\right) SMTD \qquad (A\text{-}7)\end{aligned}$$

$$\left[CO_2\right]_{th} = \frac{a\left(0.2099 - [O_2]\right)}{a + 0.1975b - 0.395c + 0.105d + e} \qquad (A\text{-}8)$$

The constants in Eq. (A-7) were obtained using the following heats of formation:

Specie	Heats of Formation, $\frac{\text{K-cal}}{\text{g-mole}}$
Fuel	ΔH^{o}_{ff}
CO	-26.42
CO_2	-94.38
NO	+21.5
SO_2	-69.3
H_2O (gas)	-57.83
H_2O (liquid)	-68.39

In the data (1) the measured levels of CO were generally less than about 200 ppm and of NO were less than about 1000 ppm. For efficiency calculations, then, both product concentrations can be neglected in Eqs. (A-5) through (A-7). For more ready use in this study, Eqs. (A-3) through (A-7) were converted to the forms

$$\text{SMTD} = \frac{\text{AKI}}{0.2099 - [CO_2] - [O_2]} \tag{A-9}$$

$$\text{AFR} = \frac{137.93}{MW_f}\left\{\text{AK2} + \left([CO_2] + [O_2]\right)\text{SMTD}\right\} \tag{A-10}$$

$$\Delta H_c = \frac{169,800}{MW_f}\left\{\text{AK3} + [CO_2]\,\text{SMTD}\right\} \tag{A-11}$$

where

$$\text{AK1} = 0.2099(0.9406b - 1.8811c + 0.5d + 4.7623e) \tag{A-12}$$

$$\text{AK2} = 0.25b - 0.5c + e \tag{A-13}$$

$$AK3 = \frac{\Delta H^o_{ff} - \frac{b}{2}\Delta H^o_{fw} + 69.3e}{94.38} \qquad \text{(A-14)}$$

and ΔH^o_{fw} = the heat of formation of water (gaseous or liquid state).

For the natural gas and low sulfur oil fuels used in the boilers (1)

	Natural Gas	Low Sulfur Oil
a	1.1103	0.5647
b	4.118	0.8622
c	0.0427	0.0034
d	0.0174	0.0014
e	0.0	0.0007
MW_f	18.41	7.742
ΔH^o_{ff}, $\frac{\text{K-cal}}{\text{g-mole}}$	-19.71	-1.25
AK1	0.7980	0.1697
AK2	1.0081	0.2146
AK3H*	1.2832	0.2997
AK3L*	1.0528	0.2514

Also, the theoretical relations of CO_2 to O_2 (under the assumption that all carbon in the fuel goes to CO_2 for the two fuels are

Natural Gas

$$\left[CO_2\right]_{th} = 0.1221 - 0.5818[O_2] \qquad \text{(A-15)}$$

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*The AK3H represents the case where the water in the combustion products is condensed (the high heat of combustion). The AK3L represents the low heat of combustion.

Low Sulfur Oil

$$\left[CO_2\right]_{th} = 0.1614 - 0.7688[O_2] \qquad \text{(A-16)}$$

If the stoichiometry developed here is used, some of the heat losses can now be calculated.

A.2 INCOMPLETE COMBUSTION

The heat input rate usually taken as a reference in combustion efficiency calculations is the high heat of combustion times the fuel flow rate. Both the high and low heats of combustion are somewhat unrealistic combustion references in that they result from simple stoichiometry and the assumption that all the carbon and hydrogen in the fuel are completely oxidized to carbon dioxide and water. Such a case is reasonably approximated in combustion systems where there are:

a. Plenty of excess air

b. Good uniform mixing

c. Sufficient time at high temperature to assure a close approach to equilibrium combustion in the initial reactions

d. Sufficiently slow cooling to assure that equilibrium is maintained throughout the cooling cycle

In the natural gas and oil diffusion flames in a utility boiler, where the reactants are simultaneously mixing, reacting and cooling, it is quite conceivable that none of these conditions are adequately satisfied.

The question of complete combustion is particularly pertinent with regard to the staged-combustion technique for NO_x control. In this technique, the initial combustion reactions take place in a fuel-rich environment. There is not enough oxygen available in the first stage to oxidize all the C to CO_2 and the H_2 to H_2O, so high concentrations of CO, free H_2 and a variety of hydrocarbon species must result. If the combustion products in this stage are cooled too much before the remaining air is mixed in (the second stage), the composition could be frozen at the first stage concentrations. The total heat then released would be much less than the high or low heat of combustion.

Figure 16 shows plots of the concentration of CO_2 in the flue gases over a range of excess oxygen levels with natural gas and oil fuels. The curves shown in the figure result from equilibrium combustion calculations

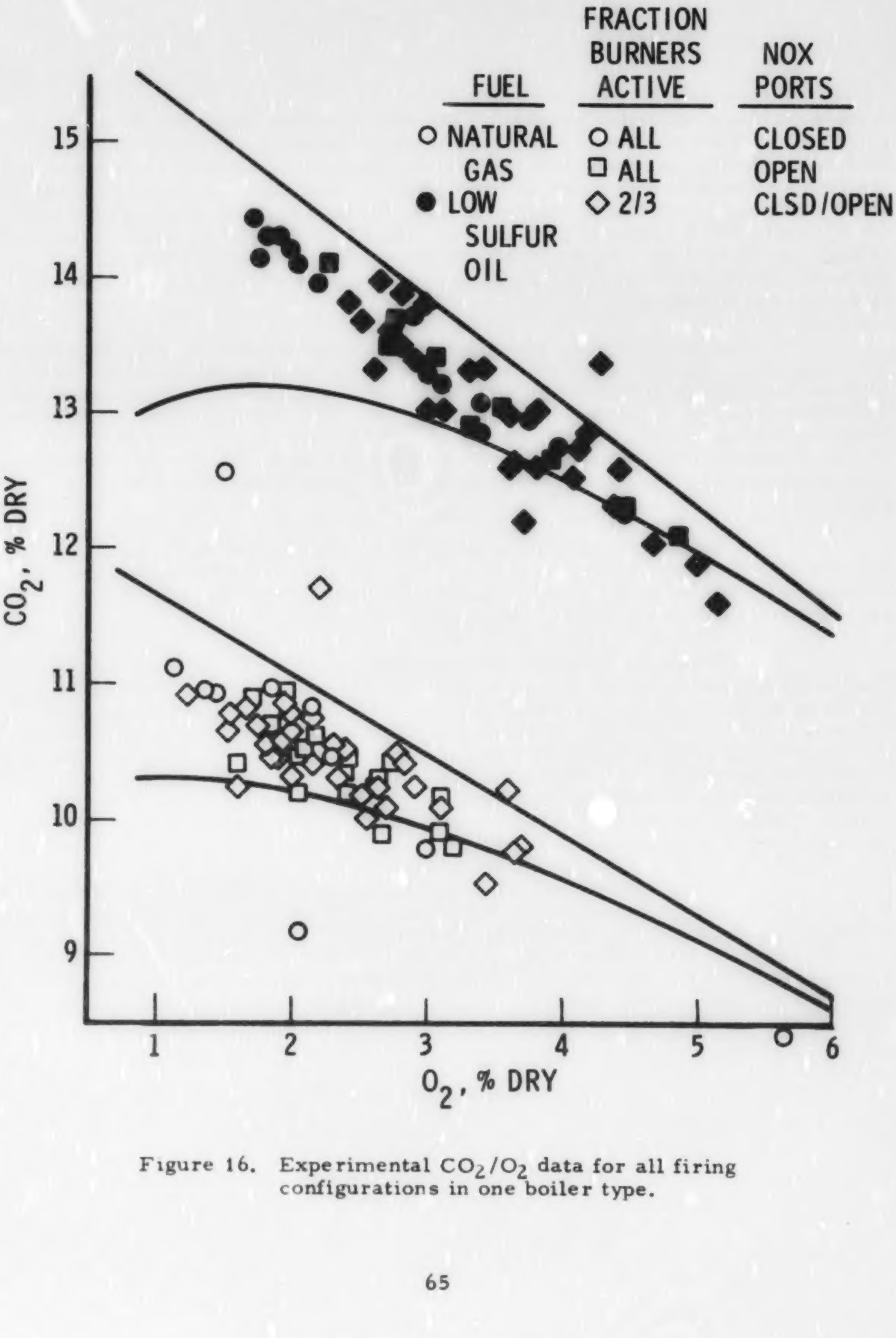

Figure 16. Experimental CO_2/O_2 data for all firing configurations in one boiler type.

using a complex computer program. The straight lines are from Eqs. (A-15) and (A-16). The equilibrium combustion calculations simulate all the combustion conditions necessary to release the low heat of combustion except that the combustion products have not been cooled. The high temperature tends to favor the more energetic products, largely through the water-gas reactions. As a result, there are less CO_2 and H_2O and more CO and H_2 (and excess O_2) at high temperatures. As the products are cooled, the CO and H_2 oxidize further to form more CO_2 and H_2O, approaching the concentrations calculated from stoichiometry.

Figure 16 also shows some of the measured CO_2/O_2 data for the tests (1). The data lie about midway between the high temperature equilibrium combustion and the stoichiometric calculations. Measured values of CO corresponding to the data are all very low. The few attempts to measure hydrocarbons also indicated very small, almost undetectable concentrations of this pollutant. Boiler operation was always adjusted (excess air) to assure negligible smoke. No other chemical species were measured in the flue gases.

Thus, it might be reasonable to assume that all the carbon in the fuel is either oxidized to CO_2 or ends up as soot or some other form of carbon, with a heat of formation of approximately zero. These are the assumptions involved in the development of Eq. (A-11).

When the actual measured CO_2 levels (expressed as mole fractions) are used, the actual heat released and available for transfer to the working fluid can be calculated from Eq. (A-11). Since the water vapor in the flue gases is clearly not condensed before the gases leave the boiler, the appropriate heat of formation of the water in Eq. (A-14) (ΔH^o_{fw}) is that of the gaseous state. The heat loss due to incomplete combustion, however, is independent of whether the water vapor condenses or not, and either the high or low heat of combustion can be used. For purposes of this calculation, the high heat of combustion is used.

If the maximum high heat of combustion (QHHCM) is defined as that released when all the carbon is oxidized to CO_2, then QHHCM can be calculated from Eq. (A-11) using measured values of O_2 and values of CO_2 concentrations calculated from Eqs. (A-15) or (A-16). The actual high heat of combustion (QHHC) can then be defined as that calculated from Eq. (A-11) using the measured concentration of CO_2. If these definitions and calculations are used, the heat loss due to incomplete combustion (QLIC) is

$$QLIC = QHHCM - QHHC \qquad (A-17)$$

The measured levels of CO_2 cannot exceed the levels shown by the straight lines in Figure 16 [Eqs. (A-15) and (A-16)] for any measured level of O_2. Regardless of any possible error in the O_2 measurement, the CO_2 level certainly cannot exceed the levels defined by the zero O_2 intercept (12.21 and 16.14 percent for natural gas and oil fuels, respectively). Flue gas analyses for CO_2 were made either by grab samples subsequently analyzed by the chemical laboratory of the utility or by continuous sampling and analysis in the Air Quality Control Mobile Laboratory of the utility. The data shown in Figure 16 are only those from analysis by the chemical laboratory. Four data points are shown where the measured CO_2 concentrations were larger than the maximum considered theoretically possible.

The data resulting from flue gas analyses conducted in the mobile, on-site laboratory, however, showed about 50 percent of the measured CO_2 levels above the maximum theoretical for both gas and oil fuels. As a result, these data were omitted from consideration in this part of the analysis, where the measured CO_2 level is critical to an estimate of the degree of incomplete combustion. For all other calculations depending on the CO_2 measurement all data were used, but if the recorded CO_2 level was above the maximum theoretical or below the equilibrium flame calculation, then the CO_2 level was taken as the average of these two calculated values. Most of the data shown in Figure 16, particularly with the natural gas fuel, are near this average. Measured O_2 levels had to be assumed accurate, but this is more likely because O_2 levels are an important boiler control parameter.

The data shown in Figure 16 represent:

a. The nominal or reference configuration with all burners active and NO_x ports (if any) closed

b. The reference case except with NO_x ports open

c. All data where one-third of the burners were operated air-only, and NO_x ports were open or closed

The latter case is represented by four boilers with four of 12 burners operated air-only and two boilers operated with eight of 24 burners air-only.

The data show that there is no apparent trend for the measured CO_2 levels to either increase or decrease as a result of these two combustion modifications. Since the heat loss due to incomplete combustion [Eqs. (A-9), (A-11), and (A-17)] is a function primarily of CO_2, this heat loss should also show no effect of these combustion modifications. Calculations of the effects of NO_x ports and one-third of the burners out of service (air-only) on efficiency losses resulting from incomplete combustion for all the data of Figure 16 show changes of about ±0.2 percent, well within the uncertainty of the data.

A.3 UNCONDENSED WATER VAPOR

Heat losses due to water in the flue gases which leaves the boiler system as uncondensed vapor are represented simply by the difference between the high and low heats of combustion. Note that ΔH_c in Eq. (A-11) represents the high or low heat of combustion, depending on the value of the constant AK3 (AK3H or AK3L); this heat loss is a constant per unit weight of fuel flow for a given fuel, independent of the values of $[CO_2]$ or SMTD. The calculation used is simply

$$\mathrm{QLH2O} = 169{,}800(\mathrm{AK3H} - \mathrm{AK3L})/\mathrm{MW}_f \qquad \text{(A-18)}$$

A.4 SENSIBLE HEAT

Since the flue gases leaving the boiler stack are relatively hot, a significant amount of the heat released by combustion leaves the boiler system in this manner, without contributing to the development of useful work. This heat loss can generally be expressed by

$$\dot{Q}_s = \dot{w}_t C_{pfg}(T_s - T_{amb}) \qquad \text{(A-19)}$$

The ambient temperature T_{amb} can be taken as 300 K (80°F), but the temperature of the gases leaving the air pre-heater (and entering the stack) was not recorded in many of the tests available to this study. Review of such data as were available indicated that the data could reasonably be fit with a function where the temperature difference $(T_s - T_{amb})$ is expressed as a linear function of boiler load (FLOAD)

$$T_s - T_{amb} = 100 + D'(\mathrm{FLOAD}) \qquad \text{(A-20)}$$

Equation (A-20) represented a good fit for all the data (at least above 50 percent of rated load) available from the Haynes boilers (1) when the constant D was equal to 71, but a value of 128 was required to fit the Scattergood data.

Similarly, data are not normally available on the specific heat of the flue gases (C_{pfg}). Equilibrium combustion calculations made at Aerospace, employing the Aerospace n-element chemistry program, were used to derive theoretical values of C_{pfg}. It was found that these values could be reasonably approximated over the range of interest by expressing C_{pfg} as a linear function of the overall boiler air-fuel ratio (AFR)

$$C_{pfg} = A' - B'(AFR) \qquad (A\text{-}21)$$

where the values of the constants A′ and B′ were found to be:

Constant	Natural Gas	Oil
A′	0.409	0.374
B′	0.00322	0.0025

Finally, the total weight flow rate of combustion products going up the stack can be expressed as

$$\dot{w}_t = \dot{w}_f \,(1 + AFR) \qquad (A\text{-}22)$$

The QLSEN is defined as the sensible heat loss per pound of fuel burned

$$QLSEN = \dot{Q}_s / \dot{w}_f \qquad (A\text{-}23)$$

Substitution of Eqs. (A-20) through (A-22) into Eq. (A-23) yields the expression used in this study to estimate sensible heat losses

$$\begin{aligned} QLSEN = {} & (1 + AFR) * (CES - CES1 * AFR) \\ & * (100 + CES2 * FLOAD) \qquad (A\text{-}24) \end{aligned}$$

where * is the FORTRAN symbol for multiply.

A.5 ELECTRICAL LOSSES

Almost nothing was available to this study to evaluate possible variations in efficiency losses in the electric generating and control equipment with operating conditions. Such losses do appear to be relatively independent of geometry and operating conditions (except load). For this study, simply to keep the overall efficiency values in reasonable perspective, a constant three percent load loss was assumed for all conditions.

A.6 EFFICIENCY CORRELATIONS

After accounting for the above four efficiency losses, the remaining efficiency represents as nearly as possible the ratio of the energy of the steam turbine shaft (output) to the heat entering the boiler steam (input). Expressed in heat units (Btu's) per second, the shaft output was given by

$$\dot{Q}_{out} = 977.1 * \text{LOAD, Btu/sec} \qquad \text{(A-25)}$$

where LOAD is in megawatts, and the numerical constant includes the three percent electrical efficiency loss described in Section A.5.

The heat entering the steam per unit of fuel flow was calculated from the initial maximum heat release (QHHCM), reduced by the three heat losses described in Sections A.2 through A.4. Thus the input heat $\dot{Q}_{in}$ was given by

$$\dot{Q}_{in} = (\text{QHHCM} - \text{QLIC} - \text{QLH2O} - \text{QLSEN}) * \text{WFT} \qquad \text{(A-26)}$$

where WFT is the fuel flow rate (and * is the FORTRAN symbol for multiply).

The so-called "steam cycle" efficiency studied here is the ratio of Eqs. (A-25) and (A-26)

$$\text{ESC} = \dot{Q}_{out}/\dot{Q}_{in} \qquad \text{(A-27)}$$

In general, it would appear that this efficiency could only be significantly affected by the steam turbine performance (involving losses such as those due to control valve throttling and off-design expansion conditions in the turbine) and by rather large shifts in the heat transfer profile through the boiler. The former effects should result largely from load variations (an operational requirement), while the latter could result from variations in the heat release rate, which in turn might result from the use of NO_x ports and the burners-out-of-service technique for NO_x reduction. Initial efforts to analyze variations in the efficiency (ESC) were directed toward identifying and correcting for the load effect in order to examine the remaining variation for a significant effect of combustion modifications made for the purpose of NO_x control.

APPENDIX B

ANALYSIS OF A COMBUSTION-AIR FEED SYSTEM COUPLED MODE OF INSTABILITY IN A UTILITY BOILER

The basic assumptions and approximations necessary to this derivation as well as schematics of the models assumed are presented in Section 3. The details of the derivation of the analytical expression for a combustion-air feed system coupled mode of instability in a utility boiler are presented in this appendix. The derivation is divided into three phases:

a. Burner air flow response to perturbations in furnace pressure at the burner exit

b. The response of furnace pressure at the burner exit to perturbations in the mass flow and air-fuel ratio issuing from the burner

c. Coupling of a. and b. into a complete combustion-air feed system coupled mode of instability

B.1 BURNER AIR FLOW RESPONSE

Referring to Figure 2, the following equations for air flow through a burner can be written:

Inlet Region

$$\overline{P}_{wb} - P_1 = R_i \dot{w}_1^2 \tag{B-1}$$

Air-only inertance

$$P_1 - P_2 = \frac{1}{2} \frac{L_b}{A_b g} \frac{d\dot{w}_1}{dt} \tag{B-2}$$

$$P_2 - P_3 = \frac{1}{2}\frac{L_b}{A_b g}\frac{d\dot{w}_b}{dt} \tag{B-3}$$

Air-only capacitance

$$\dot{w}_b = \dot{w}_1 - g\frac{A_b L_b}{a^2}\frac{dP_2}{dt} \tag{B-4}$$

Exit region (including flame)

$$P_3 - P_f = R_3\dot{w}_b^2 \tag{B-5}$$

To avoid the complications of nonlinear stability analysis, Eqs. (B-1) through (B-5) must be linearized and transformed into the LaPlace domain. This is a standard procedure in stability analysis except for the observation, first made in the previous study (1), that R_3 in Eq. (B-5) is not necessarily a constant [as is R_i in Eq. (B-1)], but may be a function of the degree of reaction which is taking place within the burner. The degree of reaction, as discussed previously (1), is in turn a function of the flow velocity, or weight flow rate, of air through the burner.

As discussed in Ref. 1, the dynamic events which take place in an active burner probably follow the sequence:

a. A small increase in furnace pressure at a burner exit causes an initial decrease in air flow velocity through the burner.

b. This decrease in flow velocity allows more of the reaction to be completed within the burner (the flame moves deeper into the burner).

c. The greater reaction within the burner increases the resistance to air flow.

d. The air flow velocity decreases even further, continuing the sequence of events.

It is not known how rapidly the flame can move in and out of the burner in response to furnace perturbations [i.e., the time delays involved in resistance (R_3) changes at constant air flow rates ($\dot{w}_b$) relative to the time delays for changes in the air flow rates ($\dot{w}_b$) at constant resistance (R_3)]. For purposes of this linear stability analysis, it is simply assumed that the burner exit resistance, like all other variables, can be expressed as the sum of a constant plus a perturbation (small) variable. Thus, to linearize Eqs. (B-1) through (B-5), the following substitutions were used:

$$P_1 = \overline{P_1} + \delta P_1 \tag{B-6}$$

$$P_2 = \overline{P_2} + \delta P_2 \tag{B-7}$$

$$P_3 = \overline{P_3} + \delta P_3 \tag{B-8}$$

$$P_f = \overline{P_f} + \delta P_f \tag{B-9}$$

$$\dot{w}_1 = \overline{\dot{w}}_1 + \delta \dot{w}_1 \tag{B-10}$$

$$\dot{w}_b = \overline{\dot{w}}_b + \delta \dot{w}_b \tag{B-11}$$

$$R_3 = \overline{R_3} + \delta R_3 \tag{B-12}$$

where δR_3 is taken as

$$\delta R_3 = \left(\frac{dR_3}{d\dot{w}_b}\right) \delta \dot{w}_b \tag{B-13}$$

Substituting Eqs. (B-6) through (B-12) into Eqs. (B-1) through (B-5), linearizing and taking the LaPlace transform results in the following

$$-\left(\delta P_1\right) = 2R_i \overline{\dot{w}}_1 \left(\delta \dot{w}_1\right) \tag{B-14}$$

$$\left(\delta P_1\right) - \left(\delta P_2\right) = \frac{1}{2} \frac{L_b}{A_b g} S\left(\delta \dot{w}_1\right) \tag{B-15}$$

$$\left(\delta P_2\right) - \left(\delta P_3\right) = \frac{1}{2}\,\frac{L_b}{A_b g}\,S\left(\delta \dot{w}_b\right) \tag{B-16}$$

$$\left(\delta \dot{w}_b\right) = \left(\delta \dot{w}_1\right) - g\,\frac{A_b L_b}{a^2}\,S\left(\delta P_2\right) \tag{B-17}$$

$$\left(\delta P_3\right) - \left(\delta P_f\right) = 2\overline{R}_3\overline{\dot{w}}_b\left(\delta \dot{w}_b\right) + \overline{\dot{w}}_b^{\,2}\left(\delta R_3\right) \tag{B-18}$$

To simplify these equations, drop the (δ) designation [keeping in mind that the variables are of perturbation (small) magnitudes], substitute Eq. (B-13) into Eq. (B-18), note that

$$\overline{\dot{w}}_1 = \overline{\dot{w}}_b \tag{B-19}$$

and let

$$L = \frac{L_b}{A_b g} \tag{B-20}$$

$$C = g\,\frac{A_b L_b}{a^2} \tag{B-21}$$

$$R_{i\ell} = 2R_i\overline{\dot{w}}_b \tag{B-22}$$

$$R_{f\ell} = 2\overline{R}_3\overline{\dot{w}}_b + \overline{\dot{w}}_b^{\,2}\,\frac{dR_3}{d\dot{w}_b} \tag{B-23}$$

Then Eqs. (B-14) through (B-18) can be written

$$-P_1 = R_{i\ell}\dot{w}_1 \tag{B-24}$$

$$P_1 - P_2 = \frac{1}{2} LS\dot{w}_1 \tag{B-25}$$

$$P_2 - P_3 = \frac{1}{2} LS\dot{w}_b \tag{B-26}$$

$$\dot{w}_b = \dot{w}_1 - CSP_2 \tag{B-27}$$

$$P_3 - P_f = R_{f\ell}\dot{w}_b \tag{B-28}$$

If simultaneous Eqs. (B-24) through (B-28) are solved and

$$T_1 = \frac{R_{i\ell}R_{f\ell}C + L}{R_{i\ell} + R_{f\ell}} \tag{B-29}$$

$$T_2 = \left(\frac{1}{2} LC\right)^{1/2} \tag{B-30}$$

$$T_3 = \left[\frac{1}{4}\left(\frac{L^2C}{R_{i\ell} + R_{f\ell}}\right)\right]^{1/3} \tag{B-31}$$

$$T_4 = R_{i\ell}C \tag{B-32}$$

the linearized burner response is

$$\left(\frac{\dot{w}_b}{P_f}\right)_{fa} = - \frac{1}{R_{i\ell} + R_{f\ell}} \frac{1 + T_4S + T_2^2S^2}{1 + T_1S + T_2^2S^2 + T_3^3S^3} \tag{B-33}$$

When there is no flame within a burner, the linearized exit resistance $R_{f\ell}$ is taken as zero, and

$$T_1 = T_{1a} = \frac{L}{R_{i\ell}} \qquad \text{(B-34)}$$

$$T_3 = T_{3a} = \left(\frac{1}{4}\ \frac{L^2C}{R_{i\ell}}\right)^{1/3} \qquad \text{(B-35)}$$

and the response of a burner becomes

$$\left(\frac{\dot{w}_b}{P_f}\right)_{nf} = -\frac{1}{R_{i\ell}}\ \frac{1 + T_4S + T_2^2S^2}{1 + T_{1a}S + T_2^2S^2 + T_{3a}^3S^3} \qquad \text{(B-36)}$$

To evaluate the frequency response of the burners, the substitution

$$S = j\omega \qquad \text{(B-37)}$$

is made, and Eq. (B-33) becomes

$$\left(\frac{\dot{w}_b}{P_f}\right)_{fa} = -\frac{1}{R_{i\ell} + R_{f\ell}}\ \frac{\left[1 - (T_2\omega)^2\right] + j(T_4\omega)}{\left[1 - (T_2\omega)^2\right] + j\left[(T_1\omega) - (T_3\omega)^3\right]} \qquad \text{(B-38)}$$

To evaluate the significance of each of the $T\omega$ terms in Eq. (B-38), a 350 MW horizontally opposed boiler was selected, represented by the following data:

L_b = 1.5 m (5 ft)

A_b = 0.5 m^2 (5.24 ft)

R_i = 0.083 $sec^2/N - m^2$ (0.0343 $sec^2/lb_f - ft^2$)

$$a = 479 \text{ m/sec } (1570 \text{ ft/sec})$$

$$\bar{\dot{w}}_b = 123 \text{ N/sec } (27.7 \text{ lb}_f\text{/sec})(\text{rated load})$$

or:

$$\bar{\dot{w}}_b = 61.6 \text{ N/sec } (13.85 \text{ lb}_f\text{/sec})(\text{half load})$$

If the maximum frequency of interest in large boilers of approximately 100 Hz (628 sec^{-1}) is considered, the maximum possible values of the $(T\omega)$ terms in Eq. (B-38) are

$$(T_1\omega)_{max} = 1.34$$

$$(T_2\omega)_{max} = 1.42$$

$$(T_3\omega)_{max} = 2.70$$

$$(T_4\omega)_{max} = 0.411$$

Thus, all of the $(T\omega)$ terms are significant (compared to 1.0) and none can be neglected. Equation (B-38), however, can be written in the form

$$\left(\frac{\dot{w}_b}{P_f}\right)_{fa} = -\frac{F_b}{R_{i\ell} + R_{f\ell}} \qquad \text{(B-39)}$$

and F_b can be written either in the complex form

$$F_b = \text{Re}(F_b) + j\text{Im}(F_b) \qquad \text{(B-40)}$$

or in the phase-amplitude form

$$F_b = \text{Mag}(F_b)e^{j\theta_b} \qquad \text{(B-41)}$$

where

$$\mathrm{Re}(F_b) = \frac{F_1}{F_3} \tag{B-42}$$

$$\mathrm{Im}(F_b) = \frac{F_2}{F_3} \tag{B-43}$$

$$\mathrm{Mag}(F_b) = \left[\frac{F_1^2 + F_2^2}{F_3^2}\right]^{1/2} \tag{B-44}$$

$$\tan(\theta_b) = \frac{F_2}{F_1} \tag{B-45}$$

$$F_1 = 1 + \left(T_1 T_4 - 2T_2^2\right)\omega^2 + \left(T_2^4 - T_3^3 T_4\right)\omega^4 \tag{B-46}$$

$$F_2 = \left[T_4 - T_1 + \left(T_1 T_2^2 - T_2^2 T_4 + T_3^3\right)\omega^2 - T_2^2 T_3^3 \omega^4\right]\omega \tag{B-47}$$

$$F_3 = 1 + \left(T_1^2 - 2T_2^2\right)\omega^2 + \left(T_2^4 - 2T_1 T_3^3\right)\omega^4 + T_3^6\omega^6 \tag{B-48}$$

At low frequencies ($\omega \rightarrow 0$), F_1 and F_3 approach 1.0 while F_2 approaches zero. Therefore, the phase lag θ_b approaches zero, the magnitude $\mathrm{Mag}(F_b)$ approaches 1.0, and Eq. (B-39) becomes

$$\left(\frac{\dot{w}_b}{P_f}\right)_{fa} = -\frac{1}{R_{i\ell} + R_{f\ell}} \tag{B-49}$$

There are three types of burners for which the low frequency response is of interest:

First, all burners when no flame (nf) is within any of the burners

$$\left(\frac{\dot{w}_b}{P_f}\right)_{nf} = -\frac{1}{2R_i\bar{\dot{w}}_{anf}} \tag{B-50}$$

then an air-only burner (a) when there is some combustion within the active burners

$$\left(\frac{\dot{w}_b}{P_f}\right)_{a} = -\frac{1}{2R_i\bar{\dot{w}}_{aa}} \tag{B-51}$$

and finally, an active, fuel-plus-air (fa) burner when there is some combustion within the active burners

$$\left(\frac{\dot{w}_b}{P_f}\right)_{fa} = -\frac{1}{2\bar{\dot{w}}_{fa}\left[R_i + \bar{R}_3 + \frac{1}{2}\bar{\dot{w}}_{fa}\left(\frac{dR3}{d\dot{w}_{fa}}\right)\right]} \tag{B-52}$$

B.2 FURNACE PRESSURE

The basic assumptions necessary to the derivation described in this section are presented and discussed in Section 5.1.2. Figure 3 in that section shows a schematic of the separation of mass flow and combustion driving forces in both time and space which is fundamental to this analysis. The basic assumptions necessary to develop a simple method to account for three-dimensional acoustics in the furnace cavity, including damping and non-perfect reflections from the solid boundaries of the furnace cavity, are listed in Section 5.1.2 as assumptions c. through f.

To reiterate briefly, these major modeling assumptions are:

a. Perturbations in mass flow and combustion rates at the exit of a burner initially generate acoustic waves of negligible pressure but finite velocity amplitudes (implies zero specific acoustic impedance).

b. Significant pressure perturbations are generated at the burner exit only by the sum of reflected acoustic waves (finite acoustic impedance).

c. The acoustic waves generated at the burner exit propagate away in equal intensity plane waves in all six directions along the three Cartesian coordinates.

d. Interactions between burners are neglected.

e. The small acoustic time delay between the burner exit and the point further out in the furnace where the concentrated combustion occurs is also neglected.

Assumptions a. and b. above are reasonable approximations of the true case. If the furnace cavity were very small, such that wave travel times throughout the cavity were very small compared to the period of the oscillations of interest ("acoustically small"), then the cavity pressure would respond almost instantly to flow and combustion variations and there would be little room for velocity oscillations (high acoustic impedance cavity). The cavity could be treated as a single lump, with pressures uniform everywhere in the cavity. This is the case most commonly treated in the literature. On the other hand, if the burner flow and combustion rate variations were occurring in an infinite (or acoustically very large) reservoir, it is quite reasonable to expect that pressure amplitudes which might be developed at the burner exit would be sufficiently small to be of little interest in stability analyses.

The accuracy of assumption c. above is not easy to evaluate. We do not know and cannot describe the complex fluid mechanical processes which generate acoustic waves from the subsonic, swirling and turbulent flow issuing from a real burner and/or from the related combustion. The local expansion of the furnace gases as a result of local combustion is undoubtedly a scalar quantity, but the burner flow rate is clearly a vector quantity of some sort which could bias the intensity of acoustic waves propagating in the various directions. Assumption c., however, not only allows a simple approximation of the coupling between the burner flow and combustion rates and the furnace cavity acoustics, but also provides a simple model of three-dimensional cavity acoustics which allows for damping during wave travel and non-perfect acoustic reflections at cavity boundaries. Considering the accuracy with which other important parameters (such as the combustion time delay and the acoustic damping and reflection coefficients) are known, assumption c. is considered a sufficiently accurate description to be useful in this analysis.

Assumptions d. and e. are simplifying approximations which should have little effect on the final results. With these five assumptions, the response of the furnace pressure at a burner exit to perturbations in air flow through an active burner can be developed.

The perfect gas law, written for a control volume at the burner exit, is

$$P_f = \left(\frac{R_o}{V_e}\right)\left(\frac{w_s T}{M}\right) \qquad \text{(B-53)}$$

The total time derivative of Eq. (B-53) is

$$\frac{1}{\bar{P}_f} \frac{dP_f}{dt} = \frac{1}{\bar{w}_s} \frac{dw_s}{dt} + \frac{1}{\bar{T}} \frac{dT}{dt} - \frac{1}{\bar{M}} \frac{dM}{dt} \qquad \text{(B-54)}$$

The last two terms in Eq. (B-54) are almost totally functions of the air-fuel ratio r during combustion. In this analysis, fuel flow rates are considered constant; therefore, air-fuel ratio variations are due to air flow rate variations alone.

Defining

$$F(r) = \frac{1}{\bar{T}} \frac{dT}{dr} - \frac{1}{\bar{M}} \frac{dM}{dr} \qquad \text{(B-55)}$$

allows Eq. (B-54) to be written as

$$\frac{1}{\bar{P}_f} \frac{dP_f}{dt} = \frac{1}{\bar{w}_s} \frac{dw_s}{dt} + \frac{F(r)}{\bar{w}_{fb}} \frac{dw_b}{dt} \qquad \text{(B-56)}$$

Since there is a time delay between the time when the unreacted gas from the burner enters the furnace and the average time when it reacts (the combustion time delay τ_c), the effect of air-fuel ratio variations on the furnace pressure at time t is the result of air flow rate variations occurring at the burner exit at an earlier time $t - \tau_c$. Thus, Eq. (B-56) should be written

$$\frac{1}{\bar{P}_f} \frac{dP_f}{dt} = \frac{1}{\bar{w}_s} \frac{dw_s}{dt} + \frac{F_{(r)}}{\bar{w}_{fb}} \frac{dw_b(t - \tau_c)}{dt} \qquad \text{(B-57)}$$

From continuity, the rate of change of the mass of gas stored in the control volume is the result of the difference in the mass flow into and out of the volume

$$\frac{dw_s}{dt} = \dot{w}_{in} - \dot{w}_{out} \qquad (B\text{-}58)$$

In the simple case of an acoustically small furnace cavity, the control volume is essentially the furnace cavity, and the only flow into the cavity is that from the burner [$\dot{w}_b$ in Eq. (B-39)]. The only flow out of the cavity is that which flows out of the radiant section (called here the furnace cavity), through the back-pass and stack to atmosphere.

This flow rate can be written as in Eq. (B-1) and linearized into the form of Eq. (B-14)

$$P_{fe} = 2R_{bp}\bar{\dot{w}}_t\dot{w}_t \qquad (B\text{-}59)$$

In Eq. (B-59), a given change in furnace pressure at the exit P_{fe} will act over the entire flow area, resulting in a relatively large change in flow out of the furnace. The rest of the analysis described herein concerns a single burner and the associated flow rates through that burner $\dot{w}_b$. For simplicity in the analysis, it will be assumed that all burners are acting in concert and that $\dot{w}_t$ of Eq. (B-59) represents an equal flow out of the control volumes at the exit of each burner. Then

$$\dot{w}_{out} = \frac{\dot{w}_t}{n_{bt}} \qquad (B\text{-}60)$$

Defining:

$$R_e = 2R_{bp}\bar{\dot{w}}_t n_{bt} \qquad (B\text{-}61)$$

the flow out of the furnace cavity can be derived from Eqs. (B-59) through (B-61)

$$\dot{w}_{out} = \frac{P_{fe}}{R_e} \qquad (B\text{-}62)$$

where R_{bp} in Eq. (B-61) can be calculated for a given boiler from data according to

$$R_{bp} = \frac{\bar{P}_{fe} - \bar{P}_{amb}}{\bar{\dot{w}}_t^2} \qquad \text{(B-63)}$$

Substituting the burner flow $\dot{w}_b$ and Eq. (B-62) into Eq. (B-58) and the result into Eq. (B-57)

$$\frac{1}{\bar{P}_f} \frac{dP_f}{dt} = \frac{1}{\bar{\dot{w}}_s} \left(\dot{w}_b - \frac{P_{fe}}{R_e} \right) + \frac{F(r)}{\bar{\dot{w}}_{fb}} \frac{d\dot{w}_b(t - \tau_c)}{dt} \qquad \text{(B-64)}$$

As long as the furnace cavity is acoustically small, variations in the burner flow and combustion rates will result immediately in cavity pressure, uniformly everywhere within the cavity, according to Eq. (B-64). The P_{fe} will always be equal to P_f. If the furnace cavity is acoustically large, however, the furnace pressure variations generated by each of the three terms on the right side of Eq. (B-64) will not occur instantly or even at the same time.

If assumptions a., b., and c. above are followed, it is assumed that variations in $\dot{w}_b$ [the first and third terms in Eq. (B-64)] generate acoustic waves, of negligible pressure but finite velocity amplitudes, which propagate spherically and uniformly away from the source. These spherical waves are here resolved into acoustic plane waves which propagate away along the positive and negative directions on the three Cartesian coordinates.

It is not necessary to write an acoustic expression for these waves. If Eq. (B-64) is written as

$$\frac{1}{\bar{P}_f} \left(\frac{dP_f}{dt} \right) = \frac{1}{\bar{P}_f} \left(\frac{dP_f}{dt} \right)_1 - \frac{1}{\bar{P}_f} \left(\frac{dP_f}{dt} \right)_2 \qquad \text{(B-65)}$$

then each of the plane waves propagating away from the source at time t can be represented by

$$\frac{1}{6\bar{P}_f} \left(\frac{dP_f}{dt} \right)_1 = \frac{1}{6} \left[\frac{\dot{w}_b}{\bar{\dot{w}}_s} + \frac{F(r)}{\bar{\dot{w}}_{fb}} \frac{d\dot{w}_b(t - \tau_c)}{dt} \right] \qquad \text{(B-66)}$$

The wave represented by Eq. (B-66) will travel through the hot gases in the furnace cavity, reflect off some solid boundary and return to the source after some time delay τ_i. During that acoustic travel it can suffer both acoustic damping and non-perfect reflections. These are accounted for here by defining the ratio of the amplitude of the returning wave to its initial amplitude as a constant K_i, where

$$K_i = K_{ri} e^{-\alpha \tau_i a} \qquad \text{(B-67)}$$

The constant K_{ri} represents a reflection coefficient and can range from zero to one. The acoustic damping coefficient in Eq. (B-67), the α in the argument of the exponent, can range from zero to small positive values.

According to assumption b. above, the pressure variations at time t in Eq. (B-66) can only be the result of burner flow and combustion rate variations which occurred at times $t - \tau_i$ earlier. Thus, (B-66) should be written

$$\frac{1}{6\bar{P}_f}\left(\frac{dP_f}{dt}\right)_1 = \frac{1}{6}\left\{K_{ri}\left[\frac{\dot{w}_b(t - \tau_i)}{\bar{w}_s} + \frac{F(r)}{\bar{w}_{fb}}\,\frac{d\dot{w}_b(t - \tau_c - \tau_i)}{dt}\right]\right\} \qquad \text{(B-68)}$$

and the total contribution of waves returning from all six directions is

$$\frac{1}{\bar{P}_f}\left(\frac{dP_f}{dt}\right)_1 = \sum_{i=1}^{6}\left\{\frac{K_{ri}}{6}\left[\frac{\dot{w}_b(t - \tau_i)}{\bar{w}_s} + \frac{F(r)}{\bar{w}_{fb}}\,\frac{d\dot{w}_b(t - \tau_c - \tau_i)}{dt}\right]\right\} \qquad \text{(B-69)}$$

The second term in Eq. (B-65) is

$$\frac{1}{\bar{P}_f}\left(\frac{P_f}{dt}\right)_2 = \frac{P_{fe}}{\bar{w}_s R_e} \qquad \text{(B-70)}$$

The pressure P_{fe} actually exists at the furnace exit some distance from the burner exit. It results from the pressure which existed at the burner exit at some time $t - \tau_e/2$ earlier, and the effect of this exit pressure on the flow out of the furnace exit will only be felt at the burner exit after a further time delay equal to $\tau_e/2$. In this case, τ_e represents the acoustic wave travel in one of the six directions discussed above (usually the long vertical distance from the burner level to the top of the furnace cavity). Equation (B-70) should be written

$$\frac{1}{\bar{P}_f}\left(\frac{dP_f}{dt}\right)_2 = \frac{P_f(t - \tau_e)}{\bar{w}_s R_e} \qquad \text{(B-71)}$$

Substituting Eqs. (B-69) and (B-71) into (B-65)

$$\frac{1}{\bar{\bar{P}}_f}\frac{dP_f}{dt} = \sum_{i=1}^{6}\left\{\frac{Kr_i}{6}\left[\frac{\dot{w}_b(t - \tau_i)}{\bar{w}_s} + \frac{F(r)}{\bar{\dot{w}}_{fb}}\frac{d\dot{w}_b(t - \tau_c - \tau_i)}{dt}\right]\right\} - \frac{P_f(t - \tau_e)}{\bar{w}_s R_e} \qquad \text{(B-72)}$$

Equation (B-72) is the expression for the variation of the furnace pressure at the burner exit in an acoustically large furnace cavity. In a smaller furnace cavity, the time delays τ_i and τ_e would decrease toward zero and Eq. (B-72) would approach Eq. (B-70). If the furnace cavity were infinitely large, Eq. (B-72) indicates that burner flow and combustion rates would not generate any pressure variations at the burner exit. These are all reasonable limits and approximations for this analysis.

Taking the LaPlace transform of Eq. (B-72) and rearranging terms

$$\left(\frac{P_f}{\dot{w}_b}\right) = R_e \frac{\left[\frac{1}{6}\sum_{i=1}^{6}\left(K_i e^{-\tau_i S}\right)\right]\left[1 + F(r)\frac{\bar{w}_s}{\bar{\dot{w}}_{fb}} S e^{-\tau_c S}\right]}{e^{-\tau_e S} + \frac{\bar{w}_s R_e}{\bar{P}_f} S} \qquad \text{(B-73)}$$

Equation (B-73) is the linearized furnace response, analogous to the burner response in Eq. (B-33).

Equation (B-73) is a rather complicated function of the complex variable S. To evaluate frequency response, the substitution of Eq. (B-37) can again be made. After much complex algebra, the furnace response again can be written either in the real and imaginary form

$$\left(\frac{P_f}{\dot{w}_b}\right) = F_F - jF_G \tag{B-74}$$

or the phase-amplitude form

$$\left(\frac{P_f}{\dot{w}_b}\right) = \mathrm{Mag}(F_p)e^{j\theta p} \tag{B-75}$$

where

$$\mathrm{Mag}(F_p) = \left(F_F^2 + F_G^2\right)^{1/2} \tag{B-76}$$

and

$$\tan(\theta_p) = - F_G/F_F \tag{B-77}$$

These are simpler forms of Eq. (B-73) except that now

$$F_F = \frac{R_e}{F_E}\left(F_A F_C - F_B F_D\right) \tag{B-78}$$

$$F_G = \frac{R_e}{F_E}\left(F_A F_D + F_B F_C\right) \tag{B-79}$$

$$F_A = F_c + F_{rr}\left[F_s \cos(\tau_c\omega) + F_c \sin(\tau_c\omega)\right] \tag{B-80}$$

$$F_B = F_s - F_{rr}\left[F_c \cos(\tau_c\omega) - F_s \sin(\tau_c\omega)\right] \tag{B-81}$$

$$F_C = \cos(\tau_e \omega) \qquad \text{(B-82)}$$

$$F_D = F_{fe} - \sin(\tau_e \omega) \qquad \text{(B-83)}$$

$$F_E = 1 - 2F_{fe} \sin(\tau_e \omega) + F_{fe}^2 \qquad \text{(B-84)}$$

$$F_c = \frac{1}{6} \sum_{i=1}^{6} K_i \cos(\tau_i \omega) \qquad \text{(B-85)}$$

$$F_s = \frac{1}{6} \sum_{i=1}^{6} K_i \sin(\tau_i \omega) \qquad \text{(B-86)}$$

$$F_{rr} = F(r) \frac{\bar{w}_s}{\bar{w}_{fb}} \omega \qquad \text{(B-87)}$$

$$F_{fe} = \frac{\bar{w}_s R_e}{\bar{P}_f} \omega \qquad \text{(B-88)}$$

Following Eqs. (B-78) through (B-88) at very low frequencies ($\omega \cong 0$)

$$F_F = K_{iavg} R_e \qquad \text{(B-89)}$$

$$F_G = 0 \qquad \text{(B-90)}$$

therefore

$$\mathrm{Mag}(F_p) = K_{iavg} R_e \qquad \text{(B-91)}$$

and

$$\theta_p = 0 \qquad \text{(B-92)}$$

B.3 OPEN LOOP RESPONSE

The overall frequency response of the open loop, as shown in Figure 4, can be obtained simply by multiplying the burner response by the furnace response

$$\frac{P_o}{P_i} = \left(\frac{\dot{w}_b}{P_f}\right)\left(\frac{P_f}{\dot{w}_b}\right) \tag{B-93}$$

If the burner response is given by Eq. (B-39), with F_b in the form given in Eq. (B-41)

$$\left(\frac{\dot{w}_b}{P_f}\right) = - \frac{\mathrm{Mag}(F_b)e^{j\theta_b}}{R_{i\ell} + R_{f\ell}} \tag{B-94}$$

and if the furnace response is given by Eq. (B-74)

$$\left(\frac{P_f}{\dot{w}_b}\right) = \mathrm{Mag}(F_p)e^{j\theta_p} \tag{B-74R}$$

then the overall open loop frequency response is given by

$$\frac{P_o}{P_i} = - \frac{\mathrm{Mag}(F_b)\mathrm{Mag}(F_p)e^{j(\theta_b+\theta_p)}}{R_{i\ell} + R_{f\ell}} \tag{B-95}$$

To be unstable, the following conditions must be met

$$\frac{\mathrm{Mag}(F_b)\mathrm{Mag}(F_p) > 1}{R_{i\ell} + R_{f\ell}} \tag{B-96}$$

and

$$\theta_b + \theta_p = (2m - 1)\pi \qquad \text{(i.e., 180 deg)} \tag{B-97}$$

where M = 0, 1, 2, ...

At very low frequencies ($\omega \cong 0$), Eq. (B-95) becomes

$$\frac{P_o}{P_i} = -\frac{K_{iavg} R_e}{R_{i\ell} + R_{f\ell}} \qquad \text{(B-98)}$$

In a typical analysis of a low-frequency, feed system coupled mode of instability in a liquid-propellant rocket engine, the following assumptions, different from those in this analysis, are usually made:

a. There is no flame in the injector orifice (burner) ($R_{f\ell} = 0$).

b. The orifice (burner) capacitance is negligible ($C = 0$).

c. The acoustic wave travel time from the region of concentrated combustion to the combustor exit is negligible ($\tau_e = 0$).

d. There is no effect of mixture ratio variations on heat release rates and subsequent pressure variations [$F(r) = 0$].

e. Wave travel times from the injector face (burner exit) to all reflecting surfaces τ_i are negligible.

f. All acoustic damping and losses due to inefficient reflections are negligible ($K_i = 1$).

g. Perturbations in mass flow from an orifice (burner) do not immediately appear as mass addition into the combustor volume (liquids) but are delayed until the concentrated reaction occurs, at a time τ_c later.

Assumptions e., f., and g. result in the modification

$$\frac{1}{6}\sum_{i=1}^{6} K_i e^{-\tau_i S} = e^{-\tau_c S} \qquad \text{(B-99)}$$

A further result of assumptions a. and b. is that

$$T_1 = \frac{L}{R_{i\ell}} \qquad \text{(B-100)}$$

and

$$T_2 = T_3 = T_4 = 0 \tag{B-101}$$

Finally, Eq. (B-33) becomes

$$\left(\frac{\dot{w}_b}{P_f}\right)_{fa} = - \frac{1}{R_{i\ell}} \frac{1}{1 + T_1 S} \tag{B-102}$$

As a result of assumptions c. through g., Eq. (B-73) becomes

$$\left(\frac{P_f}{\dot{w}_b}\right) = R_e \frac{e^{-\tau_c S}}{1 + \frac{\bar{w}_s R_e}{\bar{P}_f} S} \tag{B-103}$$

If Eqs. (B-102) and (B-103) are multiplied, the open loop response becomes

$$\frac{P_o}{P_i} = - \frac{R_e}{R_{i\ell}} \frac{e^{-\tau_c S}}{\left(1 + \frac{L}{R_{i\ell}} S\right)\left(1 + \frac{\bar{w}_s R_e}{\bar{P}_f} S\right)} \tag{B-104}$$

Equation (B-104) represents a common solution for a low-frequency, feed system coupled mode of instability in a liquid rocket engine, with only one propellant feed system involved see, for example (Ref. 5 through Ref. 7) . Thus, the solution derived for this case represents an extension of previous analyses and yields solutions typical of those previous analyses when appropriate simplifying assumptions are made.